NPS-SM-00-006

NAVAL POSTGRADUATE SCHOOL
Monterey, California

A PRELIMINARY EVALUATION OF A READINESS-BASED REPAIRABLE ITEM INVENTORY MODEL FOR THE U.S. NAVY

by

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May 2000

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Prepared for: Naval Supply Systems Command
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This report was prepared for:

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and funded by :

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REPORT DOCUMENTATION PAGE			Form Approved OMB No 0704-0188

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE May 2000	3. REPORT TYPE AND DATES COVERED Technical Report

4. TITLE AND SUBTITLE A PRELIMINARY EVALUATION OF A READINESS-BASED REPAIRABLE ITEM INVENTORY MODEL FOR THE U.S. NAVY	5. FUNDING AC8NYCKPX10112
6. AUTHOR(S) Alan W. McMasters	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Department of Systems Management Naval Postgraduate School Monterey, CA 93943-5000	8. PERFORMING ORGANIZATION REPORT NUMBER NPS-SM-00-006
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) Naval Supply Systems Command Code SUP00 5450 Carlisle Pike Mechanicsburg, PA 17055	10. SPONSORING/MONITORING AGENCY REPORT NUMBER
11. SUPPLEMENTARY NOTES	
12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release. Distribution unlimited.	12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words.)

A new wholesale level replenishment model is proposed for managing the Navy's inventories of repairable items. It is a readiness-based model which seeks to determine item depths for a weapon system which minimize the system's Mean Supply Response Time subject to budget constraint. The model incorporates both a batch procurement and batch repair of the items. Required inputs to this model are the specified values of each. The model assumes that demand is a Poisson process. The model formulation is presented. The solution procedure, which uses marginal analysis, is described. The model's performance is illustrated with an example of ten items. The results show that the proposed model provides much better Mean Supply Response Time values than the current Navy model. As an added benefit, it also gives better Supply Material Availability values that the current model. Results are also presented of a study conducted to determine potentially desirable values for the procurement order quantity and repair induction quantity. Finally, the use of a Mean Supply Response Time goal to determine the depths of the inventory is illustrated.

14. SUBJECT TERMS inventory management, inventory models, Navy repairable items, readiness-based sparing	15. NUMBER OF PAGES 83
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAR

NSN 7540-01-280-5800

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std 239-18

ABSTRACT

A new wholesale level replenishment model is proposed for managing the Navy's inventories of repairable items. It is a readiness-based model which seeks to determine the depths of items of a weapon system which minimize the system's Mean Supply Response Time subject to budget constraint. The model incorporates both a batch procurement and batch repair of the items. Required inputs to this model are the specified values of each. The model assumes that demand is a Poisson process. The model formulation is presented. The solution procedure, which uses marginal analysis, is described. The budget generation process is also described. The model's performance is illustrated with an example of ten items. The results show that the proposed model provides much better Mean Supply Response Time values than the current Navy model. As an added benefit, it also gives better Supply Material Availability values that the current model. Results are also presented of a study conducted to determine potentially desirable values for the procurement order quantity and repair induction quantity. Finally, the use of a Mean Supply Response Time goal to determine the depths of the items is illustrated.

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CHAPTER 1 - INTRODUCTION

A. Background

In the 1960's the Navy installed the first mainframe computers to manage their vast inventories of spare and repair parts. Along with these computers they installed inventory management models which had been developed by Hadley and Whitin [4]. The objective function of these models was the minimization of the average annual total variable costs to procure and hold inventories.

The Navy manages both consumable and repairable items. Consumable items are discarded when they cease to function correctly. For repairable items an attempt is made to repair a nonfunctioning item. The inventory models developed by Hadley and Whitin [4] were for the consumable items. Since there was no model in Reference [4] for repairables the Navy decided to approach the repairable problem by subdividing the problem into two distinct parts, those nonfunctioning units which couldn't be repaired and those that could be repaired. The units which could not be repaired were replaced in batches through a procurement action. Those that could be repaired were batch inducted for repair. Each part was "managed" using the same model structure as was being used for the consumable items. Using this two-part approach, the Navy was able to develop formulas for the economic order and repair quantities.

To determine the two reorder points the Navy needed to have a backorder cost. However, they had no way of determining such a cost. Therefore they adopted an approach which was to meet a certain goal for the average annual requisition fill rate. This measure was called the "Supply Material Availability"

or SMA and the goal was an average SMA of 85% over all items in a cognizance group. From the SMA goal an implied backorder cost could be determined. Using the formulas from Reference [4] for expected number of backorders at any instant of time the Navy was also able to compute an approximate average days delay for any requisition (ADD). ADD is equivalent to the mean supply response time (MSRT) used as the objective function in the development of the new wholesale provisioning model in the early 1980's.

In 1982 the Navy attempted to integrate the two parts. Unfortunately, the effort was only partially successful; two inventory management models still exist.

In the late 1970's the decision was made to upgrade the mainframe computers. The Navy decided that it would also be a good time to review its models and improve them where possible. The Naval Postgraduate School was asked to participate in this model improvement process. In 1984 the Navy accepted a wholesale provisioning model developed by Richards and McMasters [8] of the Naval Postgraduate School which had a readiness-based objective function. It was the minimization of the Mean Supply Response Time (MSRT) for a group of new items for a specific weapon system. This objective function was to be subject to a provisioning budget constraint. Unfortunately, the Navy did not have a replenishment model which was readiness-based. Therefore, the provisioning model has never been used.

In an attempt to resolve this problem this author began the search for a replenishment model in 1986 which has the same objective function and constraint as the provisioning model.

An appropriate replenishment model for managing a group of

consumable items was developed in 1989 [2]. A replenishment model for repairable items was more difficult because of the complexity of the process. In 1988 a study by this author of preliminary simulation results suggested that when demand was modeled as a Poisson process that the probability distribution for the inventory position (on-hand + on-order + in repair - backorders) at any instant of time could be approximated by the convolution of two discrete Uniform distributions, one for repairable carcasses and the other for carcasses which were either not returned or not repairable (attritions). Batch procurement of a quantity Q_P and batch induction of a repair quantity Q_R were assumed (see References [1] and [5] for the model's details).

The simulation model required further refining after discussions with Naval Supply Systems Command (NAVSUP) operations analysts and repairables managers. The current form of the simulation model for repairables was finalized in 1992. When a Poisson demand occurs the model decides if there is a carcass being returned with the demand; if so, then that carcass enters a repair queue; if not, then that information is sent to an attrition queue. When Q_R carcasses have accumulated in the repair queue the entire batch is sent to the depot for repair. However, they are usually inducted one at a time. As each is inducted it is determined whether it is capable of being repaired; if not, then an attrition is added to the attrition queue. Carcasses which can be successfully repaired ("good" carcasses) pass through the repair process. The first "good" carcass goes immediately into repair and departs a repair turnaround time (RTAT) later. The second carcass waits a short period of time, REP quarters, (as if waiting for the first item to finish the first stage of repair) and then enters the

repair process if it can be repaired; otherwise, it is rejected and recorded as an attrition and the next carcass is immediately examined. A "good" carcass completes repair RTAT quarters later. As each "good" carcass in repair is completed it is returned to the ready-for-issue (RFI) inventory. When the attrition queue reaches a size Q_P a procurement of Q_P units is made and that batch is sent to the RFI inventory a procurement lead time (PCLT quarters) later. A flow chart of this model is presented as Figure 1 in Chapter 6 of Reference [6].

The simulation model was first successfully programmed in 1993 by Maher [5]. The next step was to develop approximate equations for describing the probability distributions for the inventory position and the net inventory at any instant of time. The former was described above and is presented in Maher [3] and Baker [1]. Maher found the conjectured inventory position distribution to be quite robust. It gave excellent results for a broad range of system parameters' values as well as for the complex interactions between the procurement and repair processes.

Baker [1] was able to develop an approximate formula for the distribution of the net inventory at any instant of time based on his simulation results. The formula was actually developed by applying stochastic modeling techniques after examining the simulation results. Baker then statistically compared the formula to the simulation results and found the formula to be an excellent fit.

Once the approximate formula for the net inventory for an item was available, the formulas for the probability of being out of stock at any instant of time and the expected number of backorders at any instant of time could be derived. The details of these derivations are presented in Reference [6]. The

probability of being out of stock is needed for the determination of the Navy's measure of performance known as the Supply Material Availability (SMA). The probability of being out of stock is also used in the formula for the expected number of backorders. The latter is used for determining the Mean Supply Response Time (MSRT) for an item and determining the average annual total variable costs associated with managing that stocked item

Since the derivations of the probability of being out of stock at any instant of time and the expected number of backorders at any instant of time have been completed, a model can be developed to determine the optimal maximum inventory position for each of the repairable items in a weapon system.

B. Objectives and Scope

The primary objective of this report is to present the optimization model for determining the maximum inventory position for each of the items in a weapon system. An investment budget provides the constraint in the model. The budget constraint will be generated based on an estimate of the maximum inventory position of each item assuming the Navy's Uniform Inventory Control Program (UICP) inventory model and the repairable item inventory data for 1988 obtained from the Navy's Inventory Control Point in Mechanicsburg, Pennsylvania. The approach to solving that model will be marginal analysis.

Several versions of the optimization model will be examined because required parameters for the model are the values of Q_P and Q_R. What values should they take? Ideas from Reference [6] will be incorporated to accomplish the second objective; namely, to present a study of the impact of Q_P and Q_R on the optimization model.

In addition to the use of an optimization model, the specification of MSRT goals can be used to determined the maximum inventory position for the items in a weapon system. This process will also be demonstrated.

The only probability distribution assumed for the demand during the aggregate lead time will be the Poisson. The use of the Normal distribution will be examined in a future report.

The computer program used to generate the results appearing in this report was written in Fortran 77 and is provided in Appendix A. It was run on the IBM S/390 mainframe at the Naval Postgraduate School.

C. Preview

Chapter 2 describes the optimization problem , the procedure used to generate the budget constraint, and the marginal analysis process used in the computer program to solve the problem. It also describes the variants of the optimization model used to examine the impact of various values of Q_P and Q_R. Chapter 3 presents and discusses the results of the computer runs. Chapter 4 presents a summary, conclusions and recommendations for future studies.

CHAPTER 2 - THE READINESS-BASED INVENTORY REPLENISHMENT MODEL

A. Introduction

This chapter presents the development of a wholesale level repairable item inventory replenishment model which has a readiness-based objective function. The intent of this model is to determine the replenishment policies after items, which are part of a weapon system, have been introduced into the Navy inventories using the wholesale level initial provisioning model of Richards and McMasters [8]. The first part of the chapter defines the optimization model and describes the marginal analysis procedure which will be used to solve the model. The second part of the chapter presents a procedure for developing a budget constraint. The last part defines additional measures of effectiveness and describes the four variants of the model used to provide insight into the effects of different Q_P and Q_R values on the optimization model.

B. Mean Supply Response Time

When one speaks of readiness-based sparing the goal is to provide an inventory of parts for a weapon system which will maximize operational availability (A_o) of a weapon system where

$$A_o = \frac{MTBF}{MTBF + MTTR + MSRT}.$$

Here,

MTBF = Aggregate Mean Time between Failures,

MTTR = Aggregate Mean Time to Repair the weapon system,

MSRT = Aggregate Mean Supply Response Time.

MTBF and MTTR are measures which are part of the engineering design of the weapon system. MSRT represents the aggregate expected time (i.e., the demand weighted average time) over all items in the weapon system for the inventory management system to provide a unit to the maintenance personnel responsible for repairing the weapon system. MSRT is the only measure of readiness which can be controlled by NAVSUP.

The equation for the MSRT for a weapon system, made up of n repairable items, can be written as

$$MSRT = \frac{\sum_{i=1}^{n} D_i MSRT_i}{\sum_{i=1}^{n} D_i}. \tag{1}$$

where D_i represents the quarterly expected demand for item i. Now it turns out that

$$D_i MSRT_i = B_i(SW), \tag{2}$$

where $B_i(SW)$ represents the expected number of backorders at any instant of time for a given item i given that its maximum inventory position is SW [2], [7]. Therefore, equation (1) can be rewritten as

$$MSRT = \frac{\sum_{i=1}^{n} B_i(SW)}{\sum_{i=1}^{n} D_i}. \tag{3}$$

To maximize A_o, NAVSUP needs to minimize its contribution to A_o; namely, MSRT. It is clear, however, that if NAVSUP has an infinite amount of money to spend on spare parts then they will buy an infinite number of new units and/or repair an infinite number of damaged but repairable units to place

in inventory. This will definitely minimize the MSRT for the weapon system. However, NAVSUP does not have an infinite budget. The problem facing NAVSUP is to try stock enough units of each item in the weapon system so as to minimize a weapon system's MSRT subject to a budget constraint.

In the readiness-based initial provisioning model developed for the Navy by Richards and McMasters [8] the budget was the initial amount of money provided by Congress for a weapon system's logistical support. However, the model being described in this report is for the replenishment of spares after a weapon system has been in use for a while. The budget for the replenishment spares could be the same as the initial provisioning budget or it could be the current total Navy Stock Fund maximum value for all of the items in inventory in support of the weapon system. In this report we will assume the latter to be the budget constraint. It can be expressed as

$$\sum_{i=1}^{n} C_i SW_i(UICP) = K, \tag{4}$$

where C_i represents the unit procurement cost for item i, $SW_i(UICP)$ represents the current maximum inventory position for item i, and K represents the current total value of all the spare units of the items in the Navy's supply system in support of a weapon system.

The problem to be in addressed in this report is how to determine the set of SW_i, $i = 1, n$, which will minimize equation (3) subject to the budget constraint given by equation (4).

The optimization approach taken in Reference [8] for solving the initial provisioning problem was to use marginal analysis. That approach is also appropriate for this problem. Basically, in each step of the marginal analysis we

have currently allocated depths of $SW_i - 1$ for each item i, $i = 1,n$. We next compute the ratio

$$R(SW_i) = \frac{B(SW_i - 1) - B(SW_i)}{C_i}. \tag{5}$$

Then we will increase the number of units of item j by one unit to SW_j when

$$R(SW_j) = \underset{i=1,n}{Max}\, R(SW_i). \tag{6}$$

After making this unit increase we will reduce the remaining available budget by C_j. If, during the steps, we reach a point where some item has a C_i value which is larger than the remaining budget, we will cease to increase SW_i for that item.

In contrast to the wholesale provisioning model of Reference [8], we will need to specify the amount of both Q_P, the batch size for a procurement action, and Q_R, the quantity of carcasses which are sent to a repair depot for a given item. What values are appropriate? We will investigate the impact of several different possible values for these quantities in this report.

We need to determine the value of K for the budget constraint if we are to solve the problem. Because we will want to compare the performance of the proposed new repairable model against the current UICP model used by NAVSUP [7], we will need to determine a estimate of the value of the UICP maximum inventory position for each item we select to be in our hypothetical weapon system and use it in equation (4) to determine the value of K.

An investigation of the UICP Consolidated Stock Status Report (CSSR) for an item was conducted to determine if the current value of the item's SW could be determined from it. Unfortunately, the important data which were missing

from the CSSR were the number of new units on order and the number of carcasses in repair. That information is kept in the UICP Due-in/Due-out File but it was not provided with the CSSR. The UICP's 1988 Computation and Research Evaluation System (CARES) data tape was investigated to see if it had sufficient data to give SW values. It does list on-hand, backorders and due-ins but the files are incomplete and/or appear to contain erroneous information. The inventory position can be computed as on-hand + due-ins - backorders. However, most of the values computed for the ten items to be used in the example in this report appeared to be excessively large.

An alternative procedure for determining SW is used below. It is based on the results of the simulation study of safety stock reported in Reference [6}.

C. Development of the UICP Budget Constraint

The procedure for developing the UICP budget constraint will be to first assume that the procurement quantity and the repair quantity for an item are given by the unconstrained "optimal values" used in the UICP model [7]. These are

$$Q_P = \sqrt{\frac{8A(D-G)}{IC}} \qquad Q_R = \sqrt{\frac{8A_2 Min(D,G)}{IC_2}}, \tag{7}$$

where

D = Expected quarterly demand for an item, units/quarter;

G = Expected quarterly regeneration rate for an item, units/quarter,

C = Unit purchase cost of an item, \$/unit,

C_2 = Unit repair cost of an item, \$/unit,

A = Procurement contract cost, \$1730 (circa 1988),

A_2 =Repair Contract cost, \$730 (circa 1988),

I =Holding cost rate for a repairable, 0.21 \$/\$-year.

The values for A and A_2 were those in use in 1988 at the Mechanicsburg Inventory Control Point (then called the Ships Parts Control Center or SPCC). 1988 was the same year as the data from the UICP records used in the analyses below. The product IC represents the holding cost for one unit of a new item for one year. Similarly, IC_2 represents the holding cost for one repaired unit for one year. It should be mentioned that "I" has been fixed at 0.21 for repairables for many years.

The next step is to determine the reorder point for the procurement part of the UICP model. This is based on a formula for the probability of being out of stock [7]. It is known as RISK.

$$RISK = \frac{IC_3D}{IC_3D + 0.5\lambda * RF},$$

where

$$C_3 = \left[1 - \frac{G}{D}\right]C + \frac{G}{D}C_2,$$

and

λ = Shortage cost for a requisition for a quarter,

RF = Requisition frequency, requisitions/quarter.

The number 0.5 represents the measure of essentiality for an item. It remained the same for all 1988 items managed by SPCC.

After the RISK is calculated its value will be constrained to lie between 0.01 and 0.4 (the 1988 constraint values for SPCC).

Next, the Program Problem Variable (PPV) was calculated using the following formula [7].

$$PPV = (D - G)PCLT + G * RTAT. \tag{8}$$

PPV is the expected demand during the average lead time, L_3 where

$$L_3 = \left[1 - \frac{G}{D}\right]PCLT + \frac{G}{D}RTAT. \tag{9}$$

Note, from the definition of L_3, that $PPV = D * L_3$.

The definition of the reorder point for the procurement problem is

$$R_P = PPV + SS \tag{10}$$

where SS = Safety Stock [7].

To determine the reorder point it is necessary to decide which distribution best represents the probability distribution for the demand during the average lead time. In 1988 the UICP model used the Poisson distribution with PPV as the mean for very slow moving items. For any active items the UICP assumed the demand during lead time distribution was Normal with a mean of PPV and a standard deviation $\sigma = \sqrt{PPVar}$. $PPVar$ stands for the Program Problem Variance which was computed from a complex formula [7]. Its value in 1988 could be obtained from that year's data. However, there were analysts that believed that it gave values which were too large. That problem appears to have been resolved recently by Bissenger [2]. The analyses to be described in this report focus on the Baker model assumptions [1]; namely, demand is Poisson distributed and PCLT and RTAT are known and constant. Therefore, we will assume that demand during the average lead time, L_3, is Poisson with a mean of PPV and a standard deviation, $\sigma = \sqrt{PPV}$. We will approximate the Poisson with

a Normal having the same mean and standard deviation as the Poisson when $PPV > 50$. Admittedly, this is a somewhat arbitrary break point but it will insure that the probability of a demand of less than zero is negligible when the Normal is used to approximate the Poisson.

If the probability distribution is Poisson then the reorder point will be determined by applying the constrained RISK value to that distribution. We will obtain the value of R_P for an item by calculating the complimentary cumulative distribution function for a range of depth values and selecting the smallest one for which the probability of demand during the average lead time is $\leq 1.0 - RISK$. If the distribution is Normal then we will determine the Normal deviate, z, for which the $P(Z \geq z) \leq RISK$ where Z is the Normal random variable for the standardized Normal distribution with mean of 0.0 and standard deviation of 1.0. Then, for the Normal case, we compute $R_P = PPV + z\sqrt{PPV}$.

To determine the approximate value of SW for a given item we first need to determine the value of safety stock associated with the reorder point. We know that, for the UICP model, equation (10) provides the relationship between the reorder point and safety stock. We can then compute safety stock SS as

$$SS = R_P - PPV. \tag{11}$$

To determine the value of SW for an item when the Poisson distribution applies for demand during average lead time we make use of the results of Chapter 6 of Reference [6]. Those results were obtained from simulation studies. The best approximate formula for safety stock in Reference [6] was found to be the following:

$$SS = SW - PPV - Q_P e^{-\left\{\frac{G}{D}\right\}} - Q_R e^{-\left\{1-\frac{G}{D}\right\}} \tag{12}$$

Therefore, solving for SW gives

$$SW = SS + PPV + Q_P e^{-\left\{\frac{G}{D}\right\}} + Q_R e^{-\left\{1-\frac{G}{D}\right\}} \tag{13}$$

Equation (12) will be used to compute the value of SW_i(UICP) associated with the UICP model for each item selected for the study below.

The budget constraint value, K, for the analyses below was then computed using equation (14).

$$K = \sum_{i=1}^{n} C_i SW_i(\text{UICP}). \tag{14}$$

D. The Optimization Models

Four types of optimization models will be examined. However, the first model, Model No. 1, is not an optimization model. Its purpose is to generate the budget constraint and to provide SW(UICP) for each item and the values of the aggregate MSRT and SMA, denoted as SMAT, for the UICP data when Q_P and Q_R are "UICP optimal;" that is, are computed using equation(s) (7). The budget constraint, equation (14), is generated using these SW(UICP) values.

The second model, Model No. 2, takes the budget constraint and the Q_P and Q_R of the UICP model and applies the marginal analysis to get new SW values for the UICP model which will minimize the aggregate MSRT. Safety stocks for the SW values are then computed using equation (12). The value of R_P, the UICP procurement reorder point for an item, is computed using equation (10). The purpose of including it in the tables is merely as a basis for comparison with the UICP model results. In the new repairable item inventory model R_P is

not needed. The rule for reordering for the new model is that when the number of attritions reaches Q_P then procure an order of Q_P units; when the number of carcasses reaches Q_R send those carcasses to a depot for repair.

The corresponding values of the aggregate MSRT and SMA, denoted as SMAT, are also calculated. Finally, the unused portion budget is printed.

The value for an item's SMA (in percent) is calculated using

$$SMA = 100(1 - P_{OUT}(SW)).$$

The aggregate value, SMAT, is computed using the following formula,

$$SMAT = \frac{\sum_{i=1}^{n} D_i SMA_i}{\sum_{i=1}^{n} D_i}.$$

As a means of comparison relative to the optimization of TVC for each item, the aggregate value of TVC, denoted by TVCT, is computed for each model. The formula for TVC is

$$TVC = \frac{4(D-G)A}{Q_P} + \frac{4CRR * D * A_2}{Q_R} + IC_3 EOH + \lambda B(SW),$$

where EOH stands for "expected on-hand." From Reference [1],

$$EOH = SW - ZB - \frac{Q_p + Q_R - 2}{2} + B(SW),$$

$$ZB = PPV + G\frac{Q_R - 1}{2} REP, \tag{15}$$

where REP represents the delay between carcasses entering the repair depot. $REP = 0$ is assumed to develop the budget constraint. The first two terms of TVC are the average annual order costs for procurement and for repair, respectively. $D - G$ represents the quarterly attrition rate and $CRR * D$ represents

the quarterly rate of carcass accumulation. The last term is the average annual backorder costs. Note that this term depends on having a value for λ, the shortage cost. In the new repairable model λ is not needed since a reorder point is not needed. The purpose of λ in the UICP model was as a "knob" to adjust the value of the reorder point to achieve a desired level of SMAT. That is not a goal of the new model.

The aggregate equation for TVCT is

$$TVCT = \frac{\sum_{i=1}^{n} D_i TVC_i}{\sum_{i=1}^{n} D_i}.$$

Model No. 3 keeps the SW values determined by Model No. 2 and computes new Q_P and Q_R values which will minimize the expected total average annual variable costs (TVC) of managing the inventory of each item. Again the associated values of MSRT, SMAT, and TVCT are computed.

Model No. 4 attempts to perform a double optimization; that is, to minimize the aggregate MSRT while selecting at each step the Q_P and Q_R values which minimize each item's TVC. This model uses a search technique to find the optimal Q_P and Q_R for each marginal analysis step.

Other variants of Models Nos. 1 and 2 are then run with different Q_P and Q_R values, such as Q_P from equation (7) and $Q_R = 1$, $Q_P = D - G$ and $Q_R = CRR * D$ and Q_P and Q_R being some fraction of their UICP values based on equation (7). The purpose of these runs was to see if some Q_P and Q_R other than that computed from equation (7) would be better for the readiness-based model.

$Q_P = D - G$ and $Q_R = CRR * D$ were selected specifically because they are similar to the minimum values ($Q_P = D - G$ and $Q_R = G$ are the actual minimums) imposed by the UICP as constraints on their optimal Q_P and Q_R. The unconstrained values were given by equation (7).

The next model first uses the approach of specifying MSRT goals for each item and then an aggregate MSRT goal for a set of items. $Q_P = D - G$ and $Q_R = CRR * D$ are used in this model.

Finally, runs of Models Nos. 1 and 2 are made to show the impact of including the time delay, REP, on MSRT and SMAT.

CHAPTER 3 - RESULTS OF THE COMPUTER RUNS

A. The Sample Items

Ten items from the 7H4A cognizance group were selected to be parts of a fictitious weapon system. They are listed in Table 1 along with their parameters. These were obtained from the UICP CARES input data tape for 1988. The cognizance coding is 7H signifying a repairable shipboard item. The third position is the Item Mission Essentiality Code (IMEC). A "4" means that the item would create a loss of primary mission capability. This is the highest level of essentiality. The fourth position describes the level of demand; "A" corresponds to a requisition frequency of 3 or more requisitions per quarter. "A" items are the Navy's most active. For this cog the λ value was \$800/qtr. in 1988.

Table 1 presents the data for the ten items which are needed by the models. The first column of the tables lists the items' National Item Identification Numbers (NIIN). The items selected have a broad range of quarterly demand rates(D). It is interesting to note that the requisition frequency (RF) (requisitions/quarter) is quite close in value to D so essentially a reasonable assumption can be made that each requisition is for one unit. The models in this report focus on the units demanded rather the requisitions but because of the closeness of D and RF the results apply for requisitions as well.

A broad range of procurement costs (C) are also present in Table 1. Half of the items (the lower five in the table) are less than \$1000 per unit while the others cost \$600 and more per unit above the most expensive in the less than \$1000 group. As a consequence, it can be expected that most of the budget

constraint will be spent on the less expensive items in the process of searching for the lowest aggregate MSRT.

B. UICP Model Results

Table 2 presents an attempt to determine the actual SW values directly from the CARES data shown in the middle four columns of the table. The column headed EXP-REPAIR (which stands for "expected repair") is the product of the number of carcasses on hand which have not been repaired and the repair survival rate (RSR). This product can be viewed as a form of on-order units from a repair depot. The rest of the on-order units are a mixture of units being procured and units in repair. The number on-hand is those units ready for issue (RFI). Backorders are those units requested but their requisitions have not been filled for one reason or another. It is important to emphasize that it is possible to have both on-hand and backorders at the same time in the real world of the Navy.

The numbers in Table 2 for CARES SW seem to be either too high or too low (as in zero). The latter situation maybe due to a lack of data on the CARES tape. The former may be due to data errors or surpluses. Because of the questionable nature of these numbers, the "correct SW value" is assumed to be that which is computed using equation (13). Those values are shown in Table 3. The values of Q_P and Q_R were computed using equation (7). R_P was determined from the RISK equation results (constrained) and the Poisson probability distribution. Safety stock was computed using equation (11). Finally, The budget constraint was $1,186,928.00 and was computed using equation(14).

Table 1. Data for the Ten 7H4A Items.

NIIN	D	G	RF	CRR	RSR	PCLT	RTAT	C	C_2
000123651	15.67	3.44	15.68	0.2467	0.89	7.44	1.20	$5278.47	$302.00
000142465	13.97	12.30	13.04	.9998	.88	12.53	2.68	1635.83	428.99
000308529	3.02	2.44	3.02	.9505	.85	11.92	1.45	2831.66	750.00
000308622	5.28	4.28	5.23	.9537	.85	8.72	2.18	1595.18	630.00
000308639	3.61	2.60	3.51	.8473	.85	12.75	1.42	2316.14	385.00
000422438	29.06	21.50	26.56	.9735	.76	5.92	0.65	701.38	495.00
000455424	9.63	8.38	7.56	.9160	.95	6.89	3.73	407.59	288.00
000455633	6.34	4.94	6.26	.9167	.85	7.09	1.82	547.08	307.00
000515913	34.98	32.18	32.00	.9684	.95	10.12	0.49	956.24	655.00
000543724	17.83	14.97	15.35	.9878	.85	6.19	2.18	140.00	31.70

C. Basic Optimization Results

Table 4 presents the first results from minimizing the aggregate MSRT while keeping the Q_P and Q_R the same as Table 3. As can be seen, the aggregate MSRT value is lower than that of Table 3. In addition, the aggregate SMA (namely, SMAT) has increased over that of Table 3 and TVCT has been reduced by a small amount. The SW values did not change much for the expensive items. but it was enough to allow the cheapest item (the last on the list) to increase by 21 units and the sixth item on the list to increase by 11 units.

A run was also made for a large number of 7H4A items to see if any of the performance results observed above would be different. The 1988 CARES tape was scanned for items which did not have the following conditions; $D = 0, G = 0, D = G, PPV > 200$. There were 784 items. The total budget was \$164,325,920.00. The UICP model gave $MSRT = 4.706$ days and $SMAT = 84.21\%$. Model No 2. gave $MSRT = 1.281$ days and $SMAT = 95.19\%$ with only \$18.42 unspent out of the budget. This large group of items gave much better results than the ten used in the example for Model No. 2 since there were more alternative ways to spend the budget.

Table 5 takes a first look at changing the Q_P and Q_R values. The values of SW were fixed at their values shown in Table 4. Then, a search for the least cost values for Q_P and Q_R was conducted. The resulting values are shown in Table 5. In particular, the Q_P values are significantly larger than those provided by the UICP model. As expected, the aggregate value of TVC, TVCT, is lower than that for Table 4. However, the penalty paid is an increase in the aggregate MSRT and a decrease in SMAT.

Table 2. Maximum Inventory Position Determined from the CARES Data.

NIIN	ON-HAND	EXP-REPAIR	ON-ORDER	BACKORDERS	CARES SW
000123651	20	4	141	19	146
000412465	380	114	12	0	506
000308529	130	17	0	0	147
000308622	0	2	51	19	34
000308639	90	6	0	0	96
000422438	130	53	144	9	318
000455424	10	171	0	0	181
000455633	0	0	0	0	0
000515913	201	170	0	0	201
000543724	742	440	225	0	742

Table 3. MODEL NO. 1 - 1988 UICP Performance.

NIIN	C ($)	PPV	Q_P	Q_R	SS	R_P	SW	MSRT (days)	SMA (%)
000123651	5278.47	95.12	12	18	3	98	116	11.58	70.74
000142465	1635.83	54.01	8	28	5	59	87	2.33	91.46
000308529	2831.66	10.45	4	10	2	12	22	7.23	86.72
000308622	1595.18	18.05	6	14	3	21	35	4.84	88.51
000308639	2316.14	16.57	5	14	2	19	32	8.94	85.71
000422438	701.38	58.73	27	35	5	64	104	3.71	83.08
000455424	407.59	39.87	14	28	7	47	77	2.40	93.29
000455633	547.08	18.92	13	21	5	24	47	3.63	91.37
000515913	956.24	44.10	14	37	5	49	89	0.73	93.33
000543724	140.00	50.34	37	115	14	64	178	3.26	92.74

Aggregate Performance: MSRT = 3.810 days SMAT = 87.78 % TVCT = $6634.81

BUDGET CONSTRAINT = $1,186,928.00

Table 4. MODEL NO. 2 - Optimal SW to Minimize Aggregate MSRT for a Budget of $1,186,928.00

NIIN	C ($)	PPV	Q_P	Q_R	SS	R_P	SW	MSRT (days)	SMA (%)
000123651	5278.47	95.12	12	18	1	96	114	15.10	64.61
000142465	1635.83	54.01	8	28	4	58	86	2.88	89.86
000308529	2831.66	10.45	4	10	1	11	21	11.24	81.42
000308622	1595.18	18.05	6	14	3	21	35	4.84	88.51
000308639	2316.14	16.57	5	14	2	19	32	8.94	85.71
000422438	701.38	58.73	27	35	16	75	115	0.60	95.92
000455424	407.59	39.87	14	28	11	51	81	0.84	97.15
000455633	547.08	18.92	13	21	8	27	50	1.34	96.07
000515913	956.24	44.10	14	37	6	50	90	0.58	94.41
000543724	140.00	50.34	37	115	35	85	199	0.11	99.47

Aggregate Performance: MSRT = 3.049 days SMAT = 91.10 % TVCT = $6618.39 Budget Unspent = $9.5

Table 5. MODEL NO. 3 - Optimal SW to Minimize Aggregate MSRT with Least Cost Q_P and Q_R for a Budget of $1,186,928.00

NIIN	C ($)	PPV	Q_P	Q_R	SS	R_P	SW	MSRT (days)	SMA (%)
000123651	5278.47	95.12	27	14	-8	87	114	32.51	46.71
000142465	1635.83	54.01	16	27	1	55	86	6.57	87.96
000308529	2831.66	10.45	7	10	0	11	21	23.24	70.26
000308622	1595.18	18.05	11	14	0	18	35	13.66	76.70
000308639	2316.14	16.57	11	12	1	18	32	19.44	75.93
000422438	701.38	58.73	35	41	8	67	115	3.61	85.09
000455424	407.59	39.87	20	30	6	46	81	3.86	90.99
000455633	547.08	18.92	18	22	5	24	50	5.79	88.72
000515913	956.24	44.10	22	41	0	44	90	3.03	82.60
000543724	140.00	50.34	63	98	34	88	199	0.44	98.47

Aggregate Performance: MSRT = 7.938 days SMAT = 81.24 % TVCT = $5743.67 Budget Unspent = $9.56

Table 6 shows the results of applying both the marginal analysis for reducing MSRT and, at each step, the determination of each item's least cost Q_P and Q_R values. As was shown in Figure 25 at the end of Chapter 7 of Reference [6], as SW increases, the MSRT value for an item decreases for a while and then it starts to increase when the least cost values are used for Q_P and Q_R for each SW value. At that SW value where MSRT starts to increase, the marginal analysis is stopped. Thus, approximately \$30,000 of the budget was not used. The aggregate MSRT, SMAT, and TVCT values are worse than their values in Tables 3, 4 and 5. The conclusion from this table is that trying to optimize two different objective functions at the same time will never lead to an optimum for either model. Besides, the CPU times for such an effort can be very large if there are a large number of items in a weapon system. Therefore, we will no longer concern ourselves with least cost Q_P and Q_R as a function of SW.

D. Results of the Q_P and Q_R Study

What Q_P and Q_R should we use as input parameters fro the new model? Should the UICP values continue to be used or is there some better alternative? To answer these questions the next investigations were of different Q_P and Q_R which would remain fixed regardless of the SW values.

We considered first the case of $Q_R = 1$ with UICP value for Q_P. This case represents the situation where carcasses are immediately inducted into repair when they become available. Tables 7 and 8 provide the results. Notice that the UICP model performance and the optimal SW model give a higher MSRT that when Q_R is computed using equation (7). This is because $Q_R = 1$ for every item

Table 6. MODEL NO. 4 - Minimizing MSRT and Minimizing TVC for a Budget of $1,186,928.00

NIIN	C ($)	PPV	Q_P	Q_R	SS	R_P	SW	MSRT (days)	SMA (%)
000123651	5278.47	95.12	31	19	-9	86	119	33.94	48.34
000142465	1635.83	54.01	16	25	0	54	83	8.35	78.03
000308529	2831.66	10.45	8	11	-1	9	21	35.68	62.00
000308622	1595.18	18.05	9	12	-1	17	30	24.28	62.76
000308639	2316.14	16.57	13	14	1	18	34	24.35	73.65
000422438	701.38	58.73	24	29	0	59	93	6.50	72.57
000455424	407.59	39.87	14	21	2	42	66	7.71	82.32
000455633	547.08	18.92	17	21	3	22	47	8.22	85.02
000515913	956.24	44.10	18	33	-3	41	78	4.33	74.49
000543724	140.00	50.34	25	38	9	59	103	2.04	92.37

Aggregate Performance: MSRT = 10.634 days SMAT = 74.07 % TVCT = $6,325.17 Budget Unspent = $29,968.14

Table 7. UICP Performance when $Q_R = 1$.

NIIN	C ($)	PPV	Q_P	Q_R	SS	R_P	SW	MSRT (days)	SMA (%)
000123651	5278.47	95.12	12	1	3	98	108	8.11	74.39
000142465	1635.83	54.01	8	1	5	59	63	6.76	73.64
000308529	2831.66	10.45	4	1	2	12	15	11.40	77.79
000308622	1595.18	18.05	6	1	3	21	24	10.78	74.59
000308639	2316.14	16.57	5	1	2	19	22	13.89	75.76
000422438	701.38	58.73	27	1	5	64	78	6.17	69.57
000455424	407.59	39.87	14	1	7	47	54	5.93	82.78
000455633	547.08	18.92	13	1	5	24	31	6.32	83.06
000515913	956.24	44.10	14	1	5	49	56	2.99	73.65
000543724	140.00	50.34	37	1	14	64	81	5.45	81.26

Aggregate Performance: MSRT = 6.035 days SMAT = 75.09 % TVCT = $61918.32

Table 8. Optimal SW to Minimize Aggregate MSRT for a Budget of \$963412.44 when $Q_R = 1$.

NIIN	C (\$)	PPV	Q_P	Q_R	SS	R_P	SW	MSRT (days)	SMA (%)
000123651	5278.47	95.12	12	1	-1	94	103	17.46	57.84
000142465	1635.83	54.01	8	1	8	62	66	3.18	83.66
000308529	2831.66	10.45	4	1	1	11	14	18.11	68.69
000308622	1595.18	18.05	6	1	4	22	26	5.03	85.93
000308639	2316.14	16.57	5	1	2	19	22	13.89	75.76
000422438	701.38	58.73	27	1	17	76	89	0.71	93.44
000455424	407.59	39.87	14	1	15	55	62	0.54	97.65
000455633	547.08	18.92	13	1	9	28	35	1.35	95.07
000515913	956.24	44.10	14	1	11	55	62	0.69	91.71
000543724	140.00	50.34	37	1	29	79	96	0.12	98.67

Aggregate Performance: MSRT = 3.659 days SMAT = 87.78 % TVCT = \$61564.96 Budget Unspent = \$107.82

is a rather severe constraint. The advantage is that the budget is reduced because SW can be smaller and still provide comparable protection.

Another more general case of reduced Q_P and Q_R is when $Q_P = D - G; Q_R = CRR * D$. This case corresponds to the expected attrition rate per quarter and the expected number of carcasses received per quarter. These are similar to the minimum values (constraints) that the UICP sets on Q_P and Q_R, respectively. Figures 9 and 10 provide the results. Again, a lower budget is required. This case shows better MSRT and SMAT both before and after optimization of SW than when Q_P and Q_R are computed using equation (7)(Tables 3 and 4). Finally, we consider three fractions (0.3, 0.5, and 0.8) of the UICP Q_P and Q_R values (given by equation (7). These results are given in Tables 11 through 16. Tables 11, 13, and 15 provide the "UICP" SW results and generated the budgets. The SW values were generated in the same way as for Tables 3 and 4. Tables 12, 14, and 16 presents the SW optimizations. The budgets and MSRT values increase and SMAT decreases as Q_P and Q_R are increased. However, the value of TVCT decreases. The increase in Q_P and Q_R results in a reduction in the order costs which is more significant than the increase in backorder costs. Finally, as Q_P and Q_R increase, the values of optimal SW show more and more of a decrease in SW for the expensive items (the first 5 in each table) and a corresponding increase in SW for the inexpensive items (the last 5 in the tables).

The conclusion from this brief study of Q_P and Q_R is that values which are smaller than equation (7) but which are functions of each item's parameters will provide lower budgets while improving the aggregate MSRT and SMAT.

Table 9. UICP Performance when $Q_P = D - G$, $Q_R = CRR * D$.

NIIN	C ($)	PPV	Q_P	Q_R	SS	R_P	SW	MSRT (days)	SMA (%)
000123651	5278.47	95.12	12	4	3	98	109	9.28	72.83
000142465	1635.83	54.01	2	14	5	59	72	2.52	89.00
000308529	2831.66	10.45	1	3	2	12	15	8.35	82.24
000308622	1595.18	18.05	1	5	3	21	26	3.81	88.58
000308639	2316.14	16.57	1	3	2	19	22	8.52	82.96
000422438	701.38	58.73	8	28	5	64	89	2.11	86.28
000455424	407.59	39.87	1	9	7	47	55	1.60	93.72
000455633	547.08	18.92	1	6	5	24	29	1.80	93.09
000515913	956.24	44.10	3	34	5	49	82	0.38	95.62
000543724	140.00	50.34	3	18	14	64	81	0.12	99.03

Aggregate Performance: MSRT = 2.586 days SMAT = 89.75 % TVCT = $12250.33

Table 10. Optimal SW to Minimize Aggregate MSRT for a Budget of $1,018494.50 when $Q_P = D - G$, $Q_R = CRR * D$.

NIIN	C ($)	PPV	Q_P	Q_R	SS	R_P	SW	MSRT (days)	SMA (%)
000123651	5278.47	95.12	12	4	1	96	108	10.79	69.62
000142465	1635.83	54.01	2	14	5	59	72	2.52	89.00
000308529	2831.66	10.45	1	3	2	12	15	8.35	82.24
000308622	1595.18	18.05	1	5	3	21	26	3.81	88.58
000308639	2316.14	16.57	1	3	3	20	22	8.52	82.96
000422438	701.38	58.73	8	28	12	71	96	0.47	96.04
000455424	407.59	39.87	1	9	10	50	58	0.57	97.41
000455633	547.08	18.92	1	6	6	25	30	1.13	95.34
000515913	956.24	44.10	3	34	3	47	80	0.64	93.39
000543724	140.00	50.34	3	18	17	67	84	0.04	99.63

Aggregate Performance: MSRT = 2.365 days SMAT = 91.30 % TVCT = $12,231.16 Budget Unspent = $74.07

The "winners" in this brief analysis are $Q_P = D - G; Q_R = CRR * D$. These will be the values used for Q_P and Q_R in the remaining analyses of this report.

E. Results from Specifying MSRT Goals

We now want to consider another view of the readiness-based approach to inventory management. Typically, there is a MSRT goal established for a weapon system and depths (SW's) for each item are sought which the meet the MSRT goal while minimizing the total investment costs. If each item is stocked to just meet the goal, then each item's depth has, by definition, the least investment cost. Tables 17, 18, and 19 show the depth for the ten items when the MSRT goal is 10 days, 5 days, and 1 day, respectively. As is to be expected the investment cost goes up as the MSRT goal is reduced.

Often there is a specified MSRT goal for the weapon system which allows some items to exceed the goal if there others which will more than meet the goal. In this case, the problem solution process becomes much more complex. A computer program to solve this problem has not been written. However, to get a feel for the results of this approach, a series of different budget values can be imposed using the existing program (see Appendix A) to see the effect on the aggregate MSRT value. Figure 1 shows the results of that study. Tables 20 and 21 present the results for budgets which provided aggregate MSRT values of approximately 10 days and 5 days. Comparing these results to those of Tables 17 and 18, respectively, it is clear that the investments required for Tables 20 and 21 are lower. The reason is, of course, that the MSRT goal for each item imposes a more severe constraint than merely specifying an aggregate goal. In Tables 20 and 21 the expensive items are allowed to have much larger MSRT values than

10 and 5 days, respectively. The money saved by doing this was than spent on the less expensive items which ended up having much smaller MSRT values than 10 and 5 days.

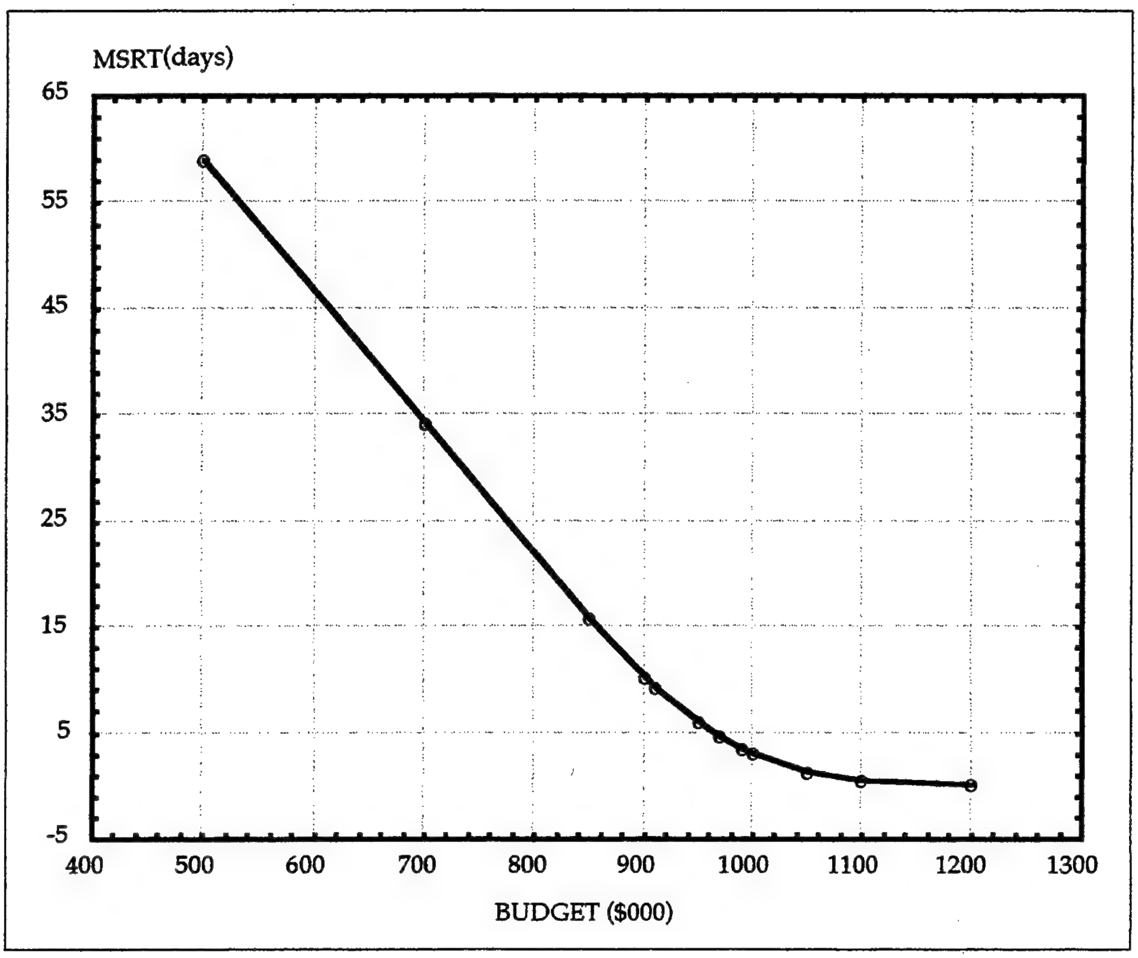

Figure 1. Aggregate MSRT as a Function of the Budget when $Q_P = D - G$ and $Q_R = CRR * D$.

Table 11. UICP Performance when $Q_P = 0.3Q_P(UICP)$, $Q_R = 0.3Q_R(UICP)$.

NIIN	C ($)	PPV	Q_P	Q_R	SS	R_P	SW	MSRT (days)	SMA (%)
000123651	5278.47	95.12	4	5	3	98	104	10.58	68.58
000142465	1635.83	54.01	2	8	5	59	67	3.01	85.62
000308529	2831.66	10.45	1	3	2	12	15	8.35	82.24
000308622	1595.18	18.05	2	4	3	21	25	5.59	84.32
000308639	2316.14	16.57	2	4	2	19	23	9.05	82.42
000422438	701.38	58.73	8	10	5	64	76	1.94	84.65
000455424	407.59	39.87	4	9	7	47	57	1.44	94.31
000455633	547.08	18.92	4	6	5	24	31	1.61	93.85
000515913	956.24	44.10	4	11	5	49	61	0.78	90.59
000543724	140.00	50.34	11	34	14	64	98	0.26	98.48

Aggregate Performance: MSRT = 2.925 days SMAT = 87.17 % TVCT = $13,901.11

Table 12. Optimal SW to Minimize Aggregate MSRT for a Budget of $959734.31 when $Q_P = 0.3Q_P(UICP)$, $Q_R = 0.3Q_R(UICP)$.

NIIN	C ($)	PPV	Q_P	Q_R	SS	R_P	SW	MSRT (days)	SMA (%)
000123651	5278.47	95.12	4	5	2	97	103	12.35	65.09
000142465	1635.83	54.01	2	8	5	59	67	3.01	85.62
000308529	2831.66	10.45	1	3	1	11	14	13.72	74.03
000308622	1595.18	18.05	2	4	4	22	26	3.67	88.85
000308639	2316.14	16.57	2	4	2	19	23	9.05	82.42
000422438	701.38	58.73	8	10	11	70	81	0.59	94.67
000455424	407.59	39.87	4	9	10	50	59	0.73	96.81
000455633	547.08	18.92	4	6	6	25	32	1.01	95.86
000515913	956.24	44.10	4	11	6	50	62	0.58	92.58
000543724	140.00	50.34	11	34	18	68	102	0.08	99.51

Aggregate Performance: MSRT = 2.737 days SMAT = 89.75 % TVCT = $13,924.54 Budget Unspent = $113.44

Table 13. UICP Performance when $Q_P = 0.5Q_P(UICP), Q_R = 0.5Q_R(UICP)$.

NIIN	C ($)	PPV	Q_P	Q_R	SS	R_P	SW	MSRT (days)	SMA (%)
000123651	5278.47	95.12	6	9	3	98	107	11.35	68.21
000142465	1635.83	54.01	4	14	5	59	73	2.62	88.98
000308529	2831.66	10.45	2	5	2	12	17	7.68	84.08
000308622	1595.18	18.05	3	7	3	21	28	4.62	87.19
000308639	2316.14	16.57	3	7	2	19	26	7.47	85.67
000422438	701.38	58.73	13	17	5	64	83	2.42	83.70
000455424	407.59	39.87	7	14	7	47	62	1.71	93.88
000455633	547.08	18.92	6	11	5	24	36	1.62	94.37
000515913	956.24	44.10	7	18	5	49	68	0.79	91.27
000543724	140.00	50.34	18	57	14	64	120	0.70	96.80

Aggregate Performance: MSRT = 3.059 days SMAT = 87.44 % TVCT = $9438.50

Table 14. Optimal SW to Minimize Aggregate MSRT for a Budget of $1,022,238.75 when $Q_P = 0.5Q_P(UICP), Q_R = 0.5Q_R(UICP)$.

NIIN	C ($)	PPV	Q_P	Q_R	SS	R_P	SW	MSRT (days)	SMA (%)
000123651	5278.47	95.12	6	9	2	97	106	13.18	64.72
000142465	1635.83	54.01	4	14	5	59	73	2.62	88.98
000308529	2831.66	10.45	2	5	1	11	16	12.49	76.84
000308622	1595.18	18.05	3	7	3	21	28	4.62	87.19
000308639	2316.14	16.57	3	7	2	19	25	11.09	80.48
000422438	701.38	58.73	13	17	13	72	91	0.41	96.50
000455424	407.59	39.87	7	14	10	50	65	0.66	97.28
000455633	547.08	18.92	6	11	7	26	37	1.05	96.07
000515913	956.24	44.10	7	18	6	50	70	0.46	94.40
000543724	140.00	50.34	18	57	20	70	127	0.13	99.27

Aggregate Performance: MSRT = 2.789 days SMAT = 90.84 % TVCT = $9513.58 Budget Unspent = $135.33

Table 15. UICP Performance when $Q_P = 0.8Q_P(UICP)$, $Q_R = 0.8Q_R(UICP)$.

NIIN	C ($)	PPV	Q_P	Q_R	SS	R_P	SW	MSRT (days)	SMA (%)
000123651	5278.47	95.12	10	14	3	98	112	11.94	68.73
000142465	1635.83	54.01	7	23	5	62	82	2.47	90.43
000308529	2831.66	10.45	3	8	2	12	20	6.72	86.72
000308622	1595.18	18.05	5	11	3	21	32	4.87	87.75
000308639	2316.14	16.57	4	11	2	19	29	9.11	84.56
000422438	701.38	58.73	21	28	5	64	96	2.81	84.64
000455424	407.59	39.87	11	23	7	47	72	1.70	94.59
000455633	547.08	18.92	10	17	5	24	42	3.04	91.78
000515913	956.24	44.10	11	30	5	49	81	0.71	93.01
000543724	140.00	50.34	29	92	14	64	155	1.90	94.37

Aggregate Performance: MSRT = 3.418 days SMAT = 87.97 % TVCT = $7101.89

Table 16. Optimal SW to Minimize Aggregate MSRT for a Budget of $1,118,985.00 when $Q_P = 0.8Q_P(UICP)$, $Q_R = 0.8Q_R(UICP)$.

NIIN	C ($)	PPV	Q_P	Q_R	SS	R_P	SW	MSRT (days)	SMA (%)
000123651	5278.47	95.12	10	14	1	96	111	13.74	65.47
000142465	1635.83	54.01	7	23	4	58	81	3.08	88.80
000308529	2831.66	10.45	3	8	1	11	19	10.74	80.94
000308622	1595.18	18.05	5	11	3	21	32	4.87	87.75
000308639	2316.14	16.57	4	11	2	19	29	9.11	84.56
000422438	701.38	58.73	21	28	14	73	104	0.63	95.36
000455424	407.59	39.87	11	23	10	50	75	0.72	97.37
000455633	547.08	18.92	10	17	7	26	44	1.48	95.40
000515913	956.24	44.10	11	30	5	49	81	0.71	93.01
000543724	140.00	50.34	29	92	26	76	167	0.22	98.97

Aggregate Performance: MSRT = 2.961 days SMAT = 90.47% TVCT = $7107.02 Budget Unspent = $63.23

Table 17. SW to Meet an MSRT Goal of 10 days for the Ten 7H Cog Items.

NIIN	C ($)	PPV	Q_P	Q_R	SW	SS	MSRT (days)	P_{OUT}	SMA (%)
000123651	5278.47	95.12	12	4	109	2	9.28	0.2717	72.83
000142465	1635.83	54.01	2	14	66	0	9.36	.2972	70.28
000308529	2831.66	10.45	1	3	15	2	8.35	.1776	82.24
000308622	1595.18	18.05	1	5	24	1	8.55	.2165	78.35
000308639	2316.14	16.57	1	3	22	3	8.51	.1704	82.96
000422438	701.38	58.73	8	28	79	-4	9.91	.4091	59.09
000455424	407.59	39.87	1	9	49	1	8.72	.2456	75.44
000455633	547.08	18.92	1	6	25	1	8.96	.2494	75.06
000515913	956.24	44.10	3	34	64	-12	9.89	.4445	55.55
000543724	140.00	50.34	3	18	64	-2	9.44	.3410	65.90

Aggregate Performance: Budget = $974,249.25 MSRT = 9.471 days SMAT = 65.45 % TVCT = $12,914.03

Table 18. SW to Meet an MSRT Goal of 5 days for the Ten 7H Cog Items.

NIIN	C ($)	PPV	Q_P	Q_R	SW	SS	MSRT (days)	P_{OUT}	SMA (%)
000123651	5278.47	95.12	12	4	113	6	4.76	0.1621	83.79
000142465	1635.83	54.01	2	14	70	3	4.07	.1596	84.03
000308529	2831.66	10.45	1	3	16	3	4.87	.1153	88.47
000308622	1595.18	18.05	1	5	26	3	3.81	.1142	88.58
000308639	2316.14	16.57	1	3	24	5	3.44	.0813	91.87
000422438	701.38	58.73	8	28	84	0	4.96	.2572	74.28
000455424	407.59	39.87	1	9	52	4	3.97	.1329	86.71
000455633	547.08	18.92	1	6	27	3	4.23	.1393	86.07
000515913	956.24	44.10	3	34	70	-6	4.50	.2745	72.55
000543724	140.00	50.34	3	18	68	1	4.40	.1965	80.35

Aggregate Performance: Budget = $1,024,681.81 MSRT = 4.473days SMAT = 79.37 % TVCT = $12572.12

Table 19. SW to Meet an MSRT Goal of 1 day for the Ten 7H Cog Items.

NIIN	C ($)	PPV	Q_P	Q_R	SW	SS	MSRT (days)	P_{OUT}	SMA (%)
000123651	5278.47	95.12	12	4	121	14	0.96	0.0329	95.97
000142465	1635.83	54.01	2	14	76	9	.86	.0359	95.42
000308529	2831.66	10.45	1	3	19	6	.74	.0125	97.66
000308622	1595.18	18.05	1	5	29	6	.93	.0219	96.53
000308639	2316.14	16.57	1	3	27	8	.70	.0123	97.92
000422438	701.38	58.73	8	28	93	9	.93	.0594	92.85
000455424	407.59	39.87	1	9	57	9	.82	.0259	96.47
000455633	547.08	18.92	1	6	51	7	.69	.0194	96.95
000515913	956.24	44.10	3	34	79	2	.81	.0661	92.01
000543724	140.00	50.34	3	18	75	8	.80	.0392	95.00

Aggregate Performance: Budget = $1,117,078.00 MSRT = 0.852 days SMAT = 94.33 % TVCT = #13,206.73

F. The Effect of REP

The final analyses examine the impact of introducing the time delay, REP, for inducting items into the repair process. Tables 22 through 25 present the changes that take place when $REP = 0.1RTAT$ and $REP = 0.2RTAT$. The values of Q_P and Q_R were computed using the UICP formulas from equation (7). These tables can therefore be compared to Tables 3 and 4. In comparing these tables the most important aspect to notice is that PPV in Tables 3 and 4 has changed to ZB which was presented in equation (15). In Tables 22 through 25 the biggest change in going from PPV to ZB is for the last item on the list (which is also the cheapest).

It is important to note that the formula for safety stock changes when $REP > 0$. Equation (12) now is modified to

$$SS = SW - ZB - Q_P e^{-\left\{\frac{G}{D}\right\}} - \frac{Q_R}{2} e^{-\left\{1-\frac{G}{D}\right\}}.$$

SW values increase when $REP > 0$ as one can see when comparing Tables 22 and 23, and Tables 24 and 25 to Tables 3 and 4. As a consequence, the budget must increase. The budget increased linearly with the percent of RTAT that was used to generate REP for each item. The MSRT values also increase while the SMAT values decline. These are not linear with the percent of RTAT but they are fairly close to being so. Recent discussions with John Boyarski, formerly with NAVSUP and now with CACI, raised questions about the usefulness of the REP term in practice. He did suggest that RTAT may be a random variable which is exponentially distributed. Dr. B. H. Bissenger, a consultant for NAVSUP, is examining the RTAT data to see what sort of distribution really fits the data.

Table 20. Optimal SW to Minimize Aggregate MSRT for a Budget of $910,000. for the Ten 7H Cog Items.

NIIN	C ($)	PPV	Q_P	Q_R	SW	SS	R_P	MSRT (days)	SMA (%)
000123651	5278.47	95.12	12	4	95	-11	84	50.18	23.63
000142465	1635.83	54.01	2	14	67	0	54	7.71	74.17
000308529	2831.66	10.45	1	3	12	0	10	32.47	52.47
000308622	1595.18	18.05	1	5	23	0	18	12.29	71.57
000308639	2316.14	16.57	1	3	19	0	17	26.58	60.29
000422438	701.38	58.73	8	28	91	7	66	1.42	89.91
000455424	407.59	39.87	1	9	55	7	47	1.60	93.72
000455633	547.08	18.92	1	6	28	4	23	2.80	90.04
000515913	956.24	44.10	3	34	75	-1	43	1.90	84.79
000543724	140.00	50.34	3	18	78	11	61	0.33	97.68

Aggregate Performance: MSRT = 9.326 days SMAT = 78.59 % TVCT = $12,103.48 Budget Unspent = $53.93

Table 21. Optimal SW to Minimize Aggregate MSRT for a Budget of $970,000. for the Ten 7H Cog Items.

NIIN	C ($)	PPV	Q_P	Q_R	SW	SS	R_P	MSRT (days)	SMA (%)
000123651	5278.47	95.12	12	4	102	-4	91	24.38	47.94
000142465	1635.83	54.01	2	14	70	3	57	4.07	84.03
000308529	2831.66	10.45	1	3	13	0	10	21.57	63.94
000308622	1595.18	18.05	1	5	25	2	20	5.79	84.03
000308639	2316.14	16.57	1	3	21	2	19	12.82	76.60
000422438	701.38	58.73	8	28	94	10	69	0.75	96.06
000455424	407.59	39.87	1	9	57	9	49	0.82	96.47
000455633	547.08	18.92	1	6	30	6	25	1.13	95.34
000515913	956.24	44.10	3	34	78	1	45	1.02	90.46
000543724	140.00	50.34	3	18	82	15	65	0.08	99.29

Aggregate Performance: MSRT = 4.696 days SMAT = 86.38% TVCT = $12000.14 Budget Unspent = $100.10

Table 22. UICP Performance with ZB Replacing PPV when $REP = 0.1RTAT$.

NIIN	C ($)	ZB	Q_P	Q_R	SS	R_P	SW	MSRT (days)	SMA (%)
000123651	5278.47	98.63	12	18	2	101	119	12.71	69.01
000142465	1635.83	98.52	8	28	6	105	133	3.49	90.18
000308529	2831.66	12.04	4	10	2	14	24	6.96	87.69
000308622	1595.18	24.12	6	14	4	28	42	4.80	89.57
000308639	2316.14	18.97	5	14	3	22	35	8.22	87.13
000422438	701.38	82.49	27	35	7	89	129	3.69	84.00
000455424	407.59	82.07	14	28	9	91	121	3.65	92.39
000455633	547.08	27.91	13	21	6	34	57	3.74	91.95
000515913	956.24	72.49	14	37	5	77	117	1.23	91.13
000543724	140.00	236.36	37	115	29	265	379	1.52	96.83

Aggregate Performance: MSRT = 4.019 days SMAT = 87.68 % TVCT = $6730.55

Table 23. Optimal SW to Minimize Aggregate MSRT for a Budget of $1,397,644.00 when $REP = 0.1RTAT$.

NIIN	C ($)	ZB	Q_P	Q_R	SS	R_P	SW	MSRT (days)	SMA (%)
000123651	5278.47	98.63	12	18	5	104	117	16.44	62.83
000142465	1635.83	98.52	8	28	19	118	132	3.49	90.18
000308529	2831.66	12.04	4	10	5	17	22	10.68	82.87
000308622	1595.18	24.12	6	14	9	33	42	4.80	89.57
000308639	2316.14	18.97	5	14	7	26	34	11.48	83.38
000422438	701.38	82.49	27	35	30	112	139	0.81	95.17
000455424	407.59	82.07	14	28	28	110	128	0.87	97.82
000455633	547.08	27.91	13	21	18	46	60	1.53	96.10
000515913	956.24	72.49	14	37	24	96	119	0.86	93.38
000543724	140.00	236.36	37	115	92	328	393	0.29	99.21

Aggregate Performance: MSRT = 3.461 days SMAT = 90.54 % TVCT = $6764.16 Budget Unspent = $57.62

Table 24. UICP Performance with ZB Replacing PPV when $REP = 0.2RTAT$.

NIIN	C ($)	ZB	Q_P	Q_R	SS	R_P	SW	MSRT (days)	SMA (%)
000123651	5278.47	102.14	12	18	3	105	123	12.17	70.23
000142465	1635.83	143.02	8	28	8	151	176	4.43	89.51
000308529	2831.66	13.64	4	10	2	16	25	6.69	88.56
000308622	1595.18	30.18	6	14	4	34	47	6.32	87.91
000308639	2316.14	21.37	5	14	3	24	37	10.51	85.00
000422438	701.38	106.25	27	35	8	114	152	3.66	84.83
000455424	407.59	124.26	14	28	11	135	161	4.52	91.97
000455633	547.08	36.90	13	21	7	44	65	3.78	92.47
000515913	956.24	100.87	14	37	6	107	146	1.31	91.53
000543724	140.00	422.37	37	115	39	461	550	1.14	97.77

Aggregate Performance: MSRT = 4.189 days SMAT = 88.04 % TVCT = $6913.54

Table 25. Optimal SW to Minimize Aggregate MSRT for a Budget of $1,610,938.00 when $REP = 0.2RTAT$.

NIIN	C ($)	ZB	Q_P	Q_R	SS	R_P	SW	MSRT (days)	SMA (%)
000123651	5278.47	102.14	12	18	5	107	121	15.79	64.22
000142465	1635.83	143.02	8	28	21	164	180	3.77	90.72
000308529	2831.66	13.64	4	10	4	18	24	14.94	78.77
000308622	1595.18	30.18	6	14	9	39	48	6.32	87.91
000308639	2316.14	21.37	5	14	8	29	37	10.51	85.00
000422438	701.38	106.25	27	35	31	137	164	0.89	95.18
000455424	407.59	124.26	14	28	32	156	174	0.94	98.02
000455633	547.08	36.90	13	21	19	56	70	1.65	96.18
000515913	956.24	100.87	14	37	25	126	149	0.94	93.52
000543724	140.00	422.37	37	115	88	510	575	1.14	97.77

Aggregate Performance: MSRT = 3.695 days SMAT = 90.51 % TVCT = $6963.44 Budget Unspent = $2.85

CHAPTER 4 - SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

A. Summary

The purpose of this report was to present the optimization model for a new readiness-based repairable item inventory management model and to show, by way of an example, the results of the optimization process using marginal analysis.

This is a companion report to Reference [6] which contains the derivations of the probability of being out of stock at any instant of time and the expected number of backorders in the system at any instant of time as well as the details of the simulation model to derive approximate formulas for safety stock. These derivations were used in the computer program (see Appendix A) to determine the optimal maximum inventory position values which minimize the aggregate mean supply response time (MSRT) for a group of items, which were assumed to represent items of a weapon system, subject to a budget constraint.

Chapter 1 reviewed the evolution of a new readiness-based repairable inventory model which is the subject of this report. Chapter 2 described the optimization model, the process used to generate the budget constraint and the optimization model variants used to study the impact of changing Q_P and Q_R. Chapter 3 presented the computer results for the optimization model and two variants of it, the impact of changing Q_P and Q_R on the optimization results, a preliminary study of specifying MSRT goals, and a brief analysis of the effect of the parameter REP on model results.

B. Conclusions

In the base case where the UICP model is compared with the new readiness-based model, the new model gave a 20% reduction in MSRT and a 4% increase in Supply Material Availability (SMA) for the 10-item sample of 7H4A cog items from the 1988 CARES input data tape.. In that case, the Q_P and Q_R values were the "UICP Optimal" and were kept the same for both models. The UICP model was used to generate the budget constraint for the new model. As expected, the new model provided a reduction in the aggregate MSRT. It was a consequence of increasing the depths of the cheaper items in the 10-item sample and reducing by minor amounts the maximum inventory values of the more expensive items of the UICP model. When the 10-item example was expanded to 784 items (which included almost all of the 7H4A items on the CARES data tape) the MSRT reduction was 73% and the SMA increase was 13%. This improving of both MSRT and SMA was first shown in the wholesale provisioning model (Reference [8]).

Using the same budget a third model kept the same maximum inventory position values of the second model but sought to determine least cost Q_P and Q_R. The model results showed an increase in MSRT over the UICP model and a decrease in SMA. Again, using the same budget, a fourth model attempted to minimize MSRT while using the least cost Q_P and Q_R at each step of the marginal analysis optimization. The process stopped long before the budget was used up because the MSRT values of all of the individual items were starting to increase. These two models show that attempting to optimize two objective functions at the same time creates an unresolvable conflict.

Since Q_P and Q_R are input parameters of the new model, what should their values be? The first application of the new model used the "UICP Optimal" values for Q_P and Q_R. Variants examined in an attempt to answer this question included the UICP minimum values, $Q_P = D - G$ and $Q_R = CRR * D$ and percentages (30%, 50%, and 80%) of the "UICP Optimal" Q_P and Q_R. As Q_P and Q_R increase there is more and more of a decrease in the maximum inventory positions for the expensive items and a corresponding increase for the inexpensive items. The budget also increases. One other variant was also examined; "UICP Optimal" Q_P and $Q_R = 1$. This corresponds to the situation where a carcass is inducted as soon as it turned in by a customer. This is typical of items which are included in the Navy's repair depot workload planning process. Both the UICP and the first version of the new model gave larger MSRT and lower SMA values because $Q_R = 1$ for all items is a severe constraint.

The conclusion from this brief study of Q_P and Q_R is that the values which are smaller than the "UICP Optimal" but which are functions of each item's parameters will provide lower budgets while improving the aggregate MSRT and SMA. The best values seem to be $Q_P = D - G$ and $Q_R = CRR * D$.

Another approach to the new model would be to set a MSRT goal rather than minimizing MSRT subject to a budget constraint. This approach can take one of two forms. The first is to specify the same goal for all items of the weapon system. The second is to specify a goal for the weapon system and allow that goal to be averaged over all the items. In this latter approach, the expensive items will probably have larger response times and the cheap items have the smaller response times than the aggregate MSRT. In the brief study conducted in

this report, setting a MSRT goal for all the repairable items in a weapon system resulted in a higher budget being required then for a specified aggregate MSRT. Finally, an another obvious result of the brief study was that the more lower the MSRT goal the larger will be the required budget.

A final analysis of this report was to examine the effect of the parameter known as REP. It is the incremental part of the repair turnaround time, which is the time for one item to move further into the repair process before the next item can enter. This might be the time required to inspect an item for the extent of repair needed or it might be the time that an item is in the first workstation. The results showed that an increase in this REP parameter resulted in an increase in the required budget.

C. Recommendations

The model studied in this report was based on the assumption that the demand during the aggregate lead time is Poisson distributed. NAVSUP personnel believe that the distribution may be Normal with a different variance than the mean in contrast to the Poisson which has its mean and variance equal. Therefore, a computer program needs to be written to allow analyses as have been done above to be done using the Normal distribution. Fortunately, the formulas for the probability of being out of stock and the expected number of backorders which are needed for that program have been derived and documented in Reference [6]. However, before a budget can be generated from the UICP model, an approximate formula for the safety stock, which is based on the Normal distribution, must be developed. This will require extensive simulation studies.

Once the safety stock formula is available, the computer program can be written to repeat the analyses presented in this report. The data which should be used in the analyses is from the 1999 CARES input files which has been provided to the author by personnel at the Naval Inventory Control Point (NAVICP). That data contains the variance of the demand during aggregate lead time based on the recent work of Bissenger [2].

A program needs to also be written which can determine a budget and the maximum inventory position values for a specified aggregate MSRT goal for both the Poisson and the Normal distributions.

A simulation study should begin which will allow RTAT to be a random variable in the repairable model. This study should provide a way to introduce a random RTAT into the distribution of the net inventory at any instant of time. It would also provide an approximate formula for safety stock.

Finally, the details of the process for implementing the new repairable inventory model need to be worked out with NAVICP personnel.

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APPENDIX A

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CC--THIS PROGRAM WAS USED TO DETERMINE THE RESULTS PRESENTED IN
CC--TABLES 3 THROUGH 25 AND FIGURE 1 OF THIS REPORT.  IT CONTAINS THE
CC--MARGINAL ANALYSIS OPTIMIZATION PROCEDURE FOR FINDING THE SET OF
CC--SW VALUES WHICH WILL MINIMIZE MSRT SUBJECT TO A BUDGET
CC--CONSTRAINT, THE BUDGET CONSTRAINT GENERATION PROCEDURE, AND THE
CC--SEARCH PROCEDURE FOR DETERMINING THE LEAST COST QP AND QR FOR A
CC--GIVEN SW VALUE NEEDED BY MODEL NO. 4.
      CHARACTER*4 COG(1000)
      CHARACTER*9 SN(1000)
      REAL PCLT(1000),D(1000),RTAT(1000),G(1000),RNUM,RDENOM,BR
      REAL C(1000),C2(1000),RSK(1000),LAM,RF(1000),C3(1000),RR(1000)
      REAL A(1000),A2(1000),CRR(1000),RSR(1000),HI,BUDGET,REP(1000)
      REAL QPU,QRU,MINRS,MAXRS,ZB(1000),PL3(1000),SSU(1000),Z(1000)
      REAL POUT(1000),EBO(1000),MSRT,MODMST,MDMSTU,GOAL
      INTEGER QP(1000),QR(1000),QPI(1000),QRI(1000),SW(1000)
      INTEGER SWI(1000),ROP(1000),STOP(1000),SS(1000),XX,MN,KK
      EXTERNAL MODMST,MDMSTU
      CHARACTER*8 NAME1(3),NAME2(3),NAME3(3),NAME4(3)
      CHARACTER*4 COGG,COG1
      DATA NAME1(1),NAME1(2),NAME1(3)/'CURRENT ','UICP*PER',*'FORMANCE'/
      DATA NAME2(1),NAME2(2),NAME2(3)/'UICP OPT','IMUM MSR','T DEPTH'/
      DATA NAME3(1),NAME3(2),NAME3(3)/'MIN COST','Q''S FOR ', 'MODEL#2'/
      DATA NAME4(1),NAME4(2),NAME4(3)/'MIN MSRT',' WITH MI','N COSTQ'/
      DATA COGG/'7H4A'/
CC--HI IS THE ANNUAL HOLDING COST RATE FOR REPAIRABLE ITEMS.
      HI=0.21
CC--THE NEXT PARAMETERS ARE THE RISK CONSTRAINTS AND SHORTAGE
CC--COST FOR 7H4A COG ITEMS.
      MINRS=0.01
      MAXRS=0.4
      LAM=800.
CC--GOAL IS USED IF A MSRT GOAL IS DESIRED FOR EACH ITEM. IF
CC--GOAL=0.0 THEN A MSRT GOAL IS NOT OF INTEREST. GOAL IS IN DAYS.
      GOAL=0.
      K=0
      N=0
    1 READ(1,10,END=11)COG1
      IF(COG1.NE.COGG)THEN
        GO TO 1
      ELSE
        I=1
        GO TO 2
      ENDIF
   10 FORMAT(A4)
    2 BACKSPACE 1
    3 READ(1,20,END=5)COG(I),SN(I),PCLT(I),RSR(I),RTAT(I),C(I),
     *D(I),G(I),RF(I),C2(I),A2(I)
   20 FORMAT(A4,1X,A9,16X,F4.2,F3.2,F4.2,4X,F10.2,10X,2F10.2,10X,F10.2,
     *101X,F10.2,F8.0)
      IF(COG(I).NE.COGG)THEN
```

```
          GO TO 3
        ELSE IF(I.LE.10)THEN
          GO TO 4
        ELSE
          GO TO 5
        ENDIF
    4   A(I)=1730.
        IF(A2(I).EQ.0.0)A2(I)=730.
CC--THE NEXT IF STATEMENTS SCREEN OUT ITEMS WHICH HAVE D=0, G=0, D=G,
CC--RSR=0 AND Z>200.  Z STANDS FOR THE UICP PROGRAM PROBLEM VARIABLE.
        IF(D(I).EQ.0.0)GO TO 3
        IF(G(I).EQ.0.0)GO TO 3
        IF(D(I).EQ.G(I))GO TO 3
        IF(RSR(I).EQ.0.0)GO TO 3
        CRR(I)=G(I)/(D(I)*RSR(I))
        IF(CRR(I).GT.1.0)CRR(I)=1.0
        PL3(I)=(1.-G(I)/D(I))*PCLT(I)+G(I)*RTAT(I)/D(I)
        Z(I)=D(I)*PL3(I)
        M=M+1
        IF(Z(I).GT.200.)GO TO 3
        K=K+1
        IF(K.LT.12)GO TO 3
CC--K IS USED TO IGNORE THE FIRST K ITEMS IN THE 7H4A FILE.
        C3(I)=(1.-G(I)/D(I))*C(I)+G(I)*C2(I)/D(I)
CC--THE UICP QP AND QR VALUES ARE COMPUTED NEXT.
        QPU=SQRT(8.*(D(I)-G(I))*A(I)/(HI*C(I)))
        QRU=SQRT(8.*MIN(D(I),G(I))*A2(I)/(HI*C2(I)))
        QPI(I)=MAX(QPU+0.5,1.)
        QRI(I)=MAX(QRU+0.5,1.)
CC--OTHER VALUES OF QP AND QR WERE ALSO USED IN THE ANALYSES.
C       QRI(I)=1
C       QPI(I)=MAX(D(I)-G(I)+0.5,1.)
C       QRI(I)=MAX(CRR(I)*D(I)+0.5,1.)
C       QPI(I)=MAX(1.0*QPU+0.5,1.)
C       QRI(I)=MAX(1.0*QRU+0.5,1.)
        QP(I)=QPI(I)
        QR(I)=QRI(I)
CC--A VALUE OF REP NEEDS TO BE SELECTED. INITIALLY REP=0.
        REP(I)=0.0
CC--REP CAN ALSO BE A PERCENTAGE OF RTAT.
C       REP(I)=0.2*RTAT(I)
CC--ZB IS BAKER'S MODIFICATION OF Z WHEN REP>0.
        ZB(I)=Z(I)+G(I)*(REAL(QR(I)-1))*REP(I)/2.
CC--RISK AND THE UICP PROCUREMENT REORDER POINT ARE COMPUTED.
        RNUM=HI*C3(I)*D(I)
        RDENOM=RNUM+LAM*RF(I)*0.5
        RISK=RNUM/RDENOM
        RSK(I)=MIN(MAXRS,MAX(MINRS,RISK))
        CALL DEPTH(RSK(I),ZB(I),ROP(I))
CC--SAFETY STOCK BASED ON THE UICP FORMULA FOR THE REORDER POINT IS
CC--ALSO NEEDED.
        SSU(I)=REAL(ROP(I))-ZB(I)
        SS(I)=SSU(I)+0.5
```

```
CC--THE FOLLOWING FORMULA IS BASED ON SIMULATION RESULTS FOR SAFETY
CC--STOCK AS A FUNCTION OF SW (THE MAX IP DEPTH) AND QP AND QR.
      SWI(I)=0.5+Z(I)+REAL(QPI(I))*EXP(-G(I)/D(I))
     *+REAL(QRI(I))*EXP(-(1.-G(I)/D(I)))+SSU(I)
      I=I+1
      GO TO 3
CC--THE BUDGET CONSTRAINT IS COMPUTED AND STOP IS INITIALIZED.
    5 BUDGET=0.0
      N=I-1
      DO 6 J=1,N
      STOP(J)=0
    6 BUDGET=BUDGET+C(J)*SWI(J)
      IF(GOAL.EQ.0.0)GO TO 9
CC--THIS PART DETERMINES SW TO MEET AN MSRT GOAL FOR EACH ITEM.
CC--IF GOAL IS ZERO THEN THIS PART IS IGNORED.
      BUDGET=0.0
      DO 7 J=1,N
      MN=SWI(J)+50
      KK=0
      DO 7 XX=1,MN
      IF(KK.GT.0)GO TO 7
      CALL TWBO(XX,QPI(J),QRI(J),ZB(J),EBO(J),POUT(J))
      MSRT=365.*EBO(J)/(4.*D(J))
      IF(MSRT.LE.GOAL)THEN
      SWI(J)=XX
      KK=KK+1
      ENDIF
    7 CONTINUE
      DO 8 J=1,N
    8 BUDGET=BUDGET+C(J)*SWI(J)
    9 CONTINUE
CC--IF A SPECIFIC BUDGET IS DESIRED, IT CAN ALSO BE ENTERED.
C     BUDGET=970000.
      WRITE(6,100)
      WRITE(6,101)COGG,N,BUDGET
      WRITE(6,102)
  100 FORMAT('1',///,'******************************************
     ************************************************')
  101 FORMAT('0',5X,'***  COG: ',A4,19X,'N: ',I4,19X,
     *'BUDGET: $',F15.2,40X,'***')
  102 FORMAT('0','********************************************
     *********************************************')
CC--THE UICP PERFORMANCE IS EVALUATED FIRST.
      CALL SSROP(N,SWI,QP,QR,D,G,Z,ZB,SS,ROP,REP)
      CALL PRTOUT(1,NAME1,BR,N,SN,SWI,ZB,QP,QR,C,C3,A,A2,D,G,CRR,
     *HI,LAM,SS,ROP,STOP)

CC--THE REST OF THE CALLS BELOW ARE NOT USED WHEN THERE IS AN
CC--MSRT GOAL.

      CALL MODOPT(N,BUDGET,MDMSTU,SW,BR,ZB,Z,C,D,RR,MR,QP,QR,
     *STOP,G,CRR,REP,C3,A,A2,LAM,HI)
      CALL SSROP(N,SW,QP,QR,D,G,Z,ZB,SS,ROP,REP)
```

```
      CALL PRTOUT(2,NAME2,BR,N,SN,SW,ZB,QP,QR,C,C3,A,A2,D,G,CRR,HI,
     * LAM,SS,ROP,STOP)

      DO 30 I=1,N
   30 CALL QPQR(SW(I),QP(I),QR(I),D(I),G(I),Z(I),ZB(I),LAM,C3(I),
     * REP(I),A(I),A2(I),CRR(I),RSR(I),HI)
      CALL SSROP(N,SW,QP,QR,D,G,Z,ZB,SS,ROP,REP)
      CALL PRTOUT(3,NAME3,BR,N,SN,SW,ZB,QP,QR,C,C3,A,A2,D,G,CRR,HI,
     * LAM,SS,ROP,STOP)

      CALL MODOPT(N,BUDGET,MODMST,SW,BR,ZB,Z,C,D,RR,MR,QP,QR,
     * STOP,G,CRR,REP,C3,A,A2,LAM,HI)
      CALL SSROP(N,SW,QP,QR,D,G,Z,ZB,SS,ROP,REP)
      CALL PRTOUT(4,NAME4,BR,N,SN,SW,ZB,QP,QR, C,C3,A,A2,D,G,CRR,HI,
     * LAM,SS,ROP,STOP)
      GO TO 12
   11 PRINT *,'THERE IS NO ITEM WITH COG=',COGG
   12 STOP
      END
C
      SUBROUTINE DEPTH(RSK,Z,ROP)
CC--THIS SUBROUTINE DETERMINES THE UICP PROCUREMENT REORDER POINT.
      REAL P(300),CP(300),RSK,CON
      INTEGER ROP,NOUT
      REAL ANORIN,X
      EXTERNAL ANORIN,UMACH
      REAL*8 ZZ
      ZZ=Z
      CON=1.-RSK
      F(Z.GT.50.)GO TO 3
      I=1
      P(I)=DEXP(-ZZ)
      CP(I)=P(I)
      IF(CP(I).LT.CON)GO TO 1
      ROP=I
      GO TO 2
    1 I=I+1
      P(I)=ZZ*P(I-1)/REAL(I-1)
      CP(I)=CP(I-1)+P(I)
      IF(CP(I).LT.CON)GO TO 1
      ROP=I
      GO TO 2
    3 CALL UMACH(2,NOUT)
      X=ANORIN(CON)
      ROP=Z+X*SQRT(Z)+0.5
    2 RETURN
      END
C
      SUBROUTINE QPQR(SW,QPP,QRR,D,G,Z,ZB,LAM,C3,REP,A,A2,CRR,RSR,HI)
CC--THIS SUBROUTINE DOES A SEARCH TO FIND THE LEAST COST QP AND
CC--QR FOR A GIVEN SW VALUE.
      REAL TVCP(1000,1000),TVCR(1000),ZB,CRR,RSR,A,A2,C3,LAM,HI,D,G
      REAL TC,APO,ARI,EOH,EBO,REP,Z
```

```
      INTEGER QP,QR,SW,QRMAX,QPMAX,X1,X2,XMAX,QPR(1000),QRR,QPP
      QPMAX=400
      QRMAX=400
      QR=0
    1 QR=QR+1
      IF(CRR.EQ.0.0.AND.QR.EQ.2)GO TO 11
      ZB=Z+G*(REAL(QR-1)*REP/2.)
      IF (QR.GT.QRMAX)GO TO 9
      QP=0
    2 QP=QP+1
      IF(CRR.EQ.1.0.AND.RSR.EQ.1.0)QP=1
      IF (QP.GT.QPMAX)GO TO 7
CC--THE FOLLOWING VARIABLES ARE BREAKPOINT VALUES FOR THE
CC--INVENTORY POSITION DISTRIBUTION.
    3 X1=MIN(QP,QR)-1
      XMAX=QP+QR-2
      X2=XMAX-X1
CC--THIS NEXT PART CALCULATES THE TIME-WEIGHTED EXPECTED NUMBER
CC--OF BACKORDERS (EBO) AND POUT.
      CALL TWBO(SW,QP,QR,ZB,EBO,POUT)
      EOH=REAL(SW)-REAL(XMAX)/2.-ZB+EBO
      APO=4.*(D-G)/REAL(QP)
      ARI=4.*CRR*D/REAL(QR)
      TC=A*APO+A2*ARI+HI*C3*EOH+LAM*EBO
      TVCP(QP,QR)=TC
      IF(CRR.EQ.1.0.AND.RSR.EQ.1.0)GO TO 4
CC--CRR=RSR=1 CORRESPONDS TO THE PURE REPAIR CASE; QP=1 AND THE
CC--SEARCH FOCUSES ON QR ONLY.  THE REST OF THE CASES NEED
CC--THE FOLLOWING STEPS.
      IF (QP.EQ.1)GO TO 2
      IF(TVCP(QP,QR).LT.TVCP(QP-1,QR))GO TO 2
      TVCR(QR)=TVCP(QP-1,QR)
      QPR(QR)=QP-1
      IF(CRR.EQ.0.0)THEN
        TVC=TVCP(QP,QR)
        QRR=QR
        QPP=QP
        GO TO 11
      ENDIF
CC--THE CASE OF CRR=0 CORRESPONDS TO THE PURE PROCUREMENT CASE.  FOR
THIS CASE QR=1 AND THE SEARCH FOCUSES ON QP ONLY.
      GO TO 5
    4 TVCR(QR)=TVCP(QP,QR)
    5 IF (QR.EQ.1)GO TO 1
      IF (TVCR(QR).LT.TVCR(QR-1))GO TO 1
      TVC=TVCR(QR-1)
      QRR=QR-1
      IF(CRR.EQ.1.0.AND.RSR.EQ.1.0)QPR(QRR)=1
    6 CONTINUE
      QPP=QPR(QRR)
      GO TO 11
    7 TVCR(QR)=TVCP(QP-1,QR)
      QPR(QR)=QP-1
```

```
      PRINT 8
    8 FORMAT(5X,'EXCEEDED QPMAX')
      TVCR(QR)=TVCP(QP-1,QR)
      GO TO 1
    9 PRINT 10
   10 FORMAT(5X,'EXCEEDED QRMAX')
   11 ZB=Z+G*(REAL(QRR-1))*REP/2.
      RETURN
      END
C
      SUBROUTINE SSROP(N,X,QP,QR,D,G,Z,ZB,SS,ROP,REP)
CC--THIS SUBROUTINE GENERATES SAFETY STOCK FOR ANY GIVEN SW, QP
CC--AND QR.
      INTEGER N,X(N),QP(N),QR(N),SS(N),ROP(N),KZ(1000)
      REAL D(N),G(N),Z(N),ZB(N),REP(N)
      DO 3 I=1,N
      IF(REP(I).EQ.0.0)GO TO 2
      SS(I)=REAL(X(I))-ZB(I)-REAL(QP(I))*EXP(-G(I)/D(I))
     *-0.5*REAL(QR(I))*EXP(-(1.-G(I)/D(I))) + 0.5
      KZ(I)=ZB(I)+0.5
      GO TO 3
    2 SS(I)=REAL(X(I))-Z(I)-REAL(QP(I))*EXP(-G(I)/D(I))
     *-REAL(QR(I))*EXP(-(1.-G(I)/D(I))) + 0.5
      KZ(I)=Z(I)+0.5
    3 ROP(I)=KZ(I)+SS(I)
      RETURN
      END
C
      SUBROUTINE MODOPT(N,B,AMODEL,X,BR,ZB,Z,C,D,RR,MR,QP,QR,
     *STOP,G,CRR, REP,C3,A,A2,LAM,HI)
CC--THIS SUBROUTINE PERFORMS THE MARGINAL ANALYSIS TO
CC--DETERMINE OPTIMAL SW FOR EACH ITEM FOR A GIVEN BUDGET.
      INTEGER N,I,K,MK,X(N),STOP(N)
      INTEGER INDEXC(1000),QP(N),QR(N)
      REAL C(N),B,BR,MR,RR(N),SR,ZB(N),D(N),G(N),CRR(N),C3(N),A(N)
      REAL A2(N),LAM,HI,Z(N),REP(N)
      SR=0.
CC--INITIALIZE SEVERAL INDICES AND THE FIRST MARGINAL ANALYSIS
CC--RATIOS.
      BR=B
      DO 2 I=1,N
      X(I)=0
CC--THE NEXT INDEX IS USED TO IDENTIFY ITEMS FOR WHICH
CC--THE BUDGET REMAINING IS LESS THAN THEIR C(I) VALUES.
      INDEXC(I)=0
CC--INITIALIZE STOP BEFORE OPTIMIZING ON SMA OR MSRT.  STOP=1
CC--MEANS THAT THE LEVEL HAS HIT THE MSRT LOWER BOUND AND STOP=2
CC--MEANS THAT MSRT IS INCREASING INSTEAD OF DECREASING
CC--AS SW (HERE X) INCREASES.
      STOP(I)=0
      RR(I)=AMODEL(ZB(I),C(I),D(I),QP(I),QR(I),X(I)+1,STOP(I),G(I),Z(I),CRR(I),
     *REP(I),C3(I),A(I),A2(I),LAM,HI)
    2 CONTINUE
```

```
    3 MK=0
      MR=-1.
      DO 4 K=1,N
        IF(STOP(K).GE.1)GO TO 4
        IF(C(K).GT.BR)INDEXC(K)=1
        IF(INDEXC(K).EQ.1)GO TO 4
        IF(RR(K) .LE. MR) GO TO 4
      MR=RR(K)
      MK=K
    4 CONTINUE
      IF(MK.EQ.0)GO TO 5
CC--ALLOCATE ONE MORE UNIT OF ITEM MK IF POSSIBLE.
      BR=BR-C(MK)
      X(MK)=X(MK)+1
      SR=MR
      RR(MK)=AMODEL(ZB(MK),C(MK),D(MK),QP(MK),QR(MK),X(MK)+1,
     *STOP(MK),G(MK),Z(MK),CRR(MK),REP(MK),C3(MK),A(MK),A2(MK),LAM,HI)
      GO TO 3
    5 RETURN
      END
C
      REAL FUNCTION MDMSTU(ZB,C,D,QP,QR,X,STOP,G,Z,CRR,REP,C3,A,
     *A2,LAM,HI)
CC--THIS SUBROUTINE DETERMINES THE MARGINAL ANALYSIS RATIO FOR
CC--MSRT FOR SPECIFIED QP AND QR AND DEPTH X (WHICH IS REALLY SW).
      REAL ZB,C,MSRT,EBOX,EBOY,POUT,TVC,D
      INTEGER X,STOP,QP,QR
      CALL TWBO(X,QP,QR,ZB,EBOX,POUT)
      CALL TWBO(X-1,QP,QR,ZB,EBOY,POUT)
      MDMSTU=(EBOY-EBOX)/C
      MSRT=365.*EBOX/(4.*D)
      IF(MSRT.LT.0.001)STOP=1
      RETURN
      END
C
      REAL FUNCTION MODMST(ZB,C,D,QP,QR,X,STOP,G,Z,CRR,REP,C3,A,
     *A2,LAM,HI)
CC--THIS SUBROUTINE DETERMINES THE MARGINAL ANALYSIS RATIO FOR
CC--MSRT FOR LEAST COST QP AND QR. IT CAN BE EXPECTED THAT THE
CC--CHANGE IN MSRT WILL GO POSITIVE AFTER A CERTAIN X (SW) VALUE.
CC--AT THAT POINT THE PROCESS WILL STOP.
      REAL ZB,C,MSRT,EBOX,EBOY,POUT,TVC,D,Z
      INTEGER X,STOP,QP,QR,QPP,QRR,QPP2,QRR2,QPP3,QRR3
      CALL QPQR(X,QPP,QRR,D,G,Z,ZB,LAM,C3,REP,A,A2,CRR,RSR,HI)
      CALL TWBO(X,QPP,QRR,ZB,EBOX,POUT)
      CALL QPQR(X-1,QPP2,QRR2,D,G,Z,ZB,LAM,C3,REP,A,A2,CRR,RSR,HI)
      CALL TWBO(X-1,QPP2,QRR2,ZB,EBOY,POUT)
      CALL QPQR(X+1,QPP3,QRR3,D,G,Z,ZB,LAM,C3,REP,A,A2,CRR,RSR,HI)
      CALL TWBO(X+1,QPP3,QRR3,ZB,EBOZ,POUT)
      QP=QPP
      QR=QRR
      IF(STOP.EQ.2)GO TO 4
```

```
CC--STOP=2 MEANS THAT AN INCREASE IN X WILL RESULT IN AN INCREASE IN
CC--MSRT. HOWEVER, DUE TO ROUNDOFF ERRORS IT IS POSSIBLE THAT SMALL
CC--EBO VALUES WILLOSCILLATE SLIGHTLY WITH SW AS THEY APPROACH
CC--ZERO. THIS LOOK-BACK AND LOOK-AHEAD IS DESIGNED TO COMPENSATE
CC--FOR THAT.
      IF(EBOX.GT.EBOY.AND.EBOZ.GT.EBOY)THEN
       STOP=2
       GO TO 4
      ELSE IF(EBOX.GT.EBOY.AND.EBOZ.LE.EBOY)THEN
       GO TO 3
      ELSE
       GO TO 2
      ENDIF
    2 MODMST=(EBOY-EBOX)/C
      MSRT=365.*EBOX/(4.*D)
      IF(MSRT.LT.0.001)STOP=1
      GO TO 4
    3 DEBO=(EBOY-EBOZ)/2.
      MODMST=DEBO/C
      MSRT=365.*(EBOY-DEBO)/(4.*D)
      IF(MSRT.LT.0.001)STOP=1
    4 RETURN
      END
C
      SUBROUTINE PRTOUT(MD,NAME,BR,N,SN,X,ZB,QP,QR,C,C3,A,A2,D,G,
     *CRR,HI,LAM,SS,ROP,STOP)
CC--THIS SUBROUTINE PRINTS OUT THE TABLE OF RESULTS.
      INTEGER X(N),MD,STOP(N),QP(N),QR(N),SS(N),ROP(N)
      REAL D(N),G(N),LAM,C3(N),A(N),A2(N),CRR(N),HI
      REAL C(N),BR,ZB(N),WSMA,WMSRT,WTVC,MSRT(1000),SMA(1000)
      REAL EBO(1000),POUT(1000),TVC(1000)
      CHARACTER*8 NAME(3)
      CHARACTER*9 SN(1000)
      WRITE(6,11)
      WRITE(6,12) MD,NAME,BR
      WRITE(6,15)
      CALL OBJECT(X,N,ZB,D,G,CRR,QP,QR,EBO,POUT,MSRT,SMA,TVC,WSMA,
     *WMSRT,WTVC,LAM,A,A2,C3,HI)
      DO 2 I=1,N
    2 WRITE(6,10)SN(I),X(I),QP(I),QR(I),SS(I),ROP(I),
     *MSRT(I),SMA(I),C(I),ZB(I),STOP(I)
      WRITE(6,13) WMSRT,WSMA,WTVC
      WRITE(6,14)
   10 FORMAT(3X,A9,4X,I5,4X,I4,4X,I4,4X,I4,4X,I4,
     *3X,F6.2,5X,F6.2,3X,F8.2,3X,F6.2,3X,I4)
   11 FORMAT('1','****************************************
     ***********************************************')
   12 FORMAT('0',1X,'MODEL (',I1,') ',3A8,6X,'BUDGET LEFT: $',F10.2)
   13 FORMAT('0',1X,'OVERALL PERFORMANCE:',' MSRT=',F8.4,'DAYS',
     *10X,'SMA=',F5.2,'%',10X,'TVC= $',F10.2)
   14 FORMAT('-','****************************************
     ***********************************************')
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   15 FORMAT('0',4X,'NIIN',8X,'DEPTH',5X,'QP',6X,'QR',6X,'SS',5X,'ROP',
     * 2X,'MSRT(DAYS)',2X,'SMA(%)',2X,'UNIT COST',3X,'PPV-B',4X,'BD CODE')
      RETURN
      END
C
      SUBROUTINE OBJECT(X,N,ZB,D,G,CRR,QP,QR,EBO,POUT,MSRT,
     * SMA,TVC,WSMA,WMSRT,WTVC,LAM,A,A2,C3,HI)
CC--THIS SUBROUTINE COMPUTES THE AGGREGATE MEASURES OF
CC--EFFECTIVENESS FOR N ITEMS FOR SPECIFIED X (WHICH IS SW) AND
CC--QP AND QR VALUES.
      INTEGER N,X(N),QP(N),QR(N)
      REAL ZB(N),SMA(N),MSRT(N),TVC(N),SD,WMSRT,WSMA,WTVC
      REAL EBO(N),POUT(N)
      REAL D(N),G(N),LAM,C3(N),A(N),A2(N),CRR(N),HI
      TSMA=0.0
      TMST=0.0
      TTVC=0.0
      SD=0.0
      DO 2 I=1,N
      SD=SD+D(I)
      CALL EBOPO(ZB(I),QP(I),QR(I),X(I),EBO(I),POUT(I),TVC(I),D(I),G(I),CRR(I),
     * LAM,A(I),A2(I),C3(I),HI)
      MSRT(I)=365.*EBO(I)/(4.*D(I))
      SMA(I)=100.*(1.-POUT(I))
      TSMA=TSMA+SMA(I)*D(I)
      TMST=TMST+MSRT(I)*D(I)
      TTVC=TTVC+TVC(I)*D(I)
    2 CONTINUE
      WSMA=TSMA/SD
      WMSRT=TMST/SD
      WTVC=TTVC/SD
      RETURN
      END
C
      SUBROUTINE EBOPO(ZB,QP,QR,SW,EBO,POUT,TVC,D,G,CRR,
     * LAM,A,A2,C3,HI)
CC--THIS SUBROUTINE COMPUTES THE MEASURES OF EFFECTIVENESS FOR
CC--A GIVEN ITEM FOR A GIVEN SW, QP AND QR.
      INTEGER QP,QR,SW
      REAL ZB,EBO,POUT,TVC
      REAL D,G,LAM,C3,A,A2,CRR,HI
CC--THIS NEXT STEP CALCULATES THE TIME-WEIGHTED EXPECTED NUMBER
CC--OF BACKORDERS (EBO) AND POUT.
      CALL TWBO(SW,QP,QR,ZB,EBO,POUT)
CC--THE TIME-WEIGHTED EXPECTED NUMBER OF UNITS ON HAND, EOH, IS
CC--CALCULATED BASED ON THE DEFINITION OF NET INVENTORY
CC--(EOH-EBO).  THE EXPECTED NET INVENTORY FORMULA IS FROM
CC--BAKER'S THESIS.
      EOH=REAL(SW)-REAL(QP+QR-2)/2.-ZB+EBO
      SMA=100*(1.-POUT)
CC--THE EXPECTED ANNUAL NUMBER OF PROCUREMENT ORDERS, APO, AND
CC--REPAIR INDUCTIONS, ARI, ARE COMPUTED NEXT.
      APO=4.*(D-G)/REAL(QP)
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      ARI=4.*CRR*D/REAL(QR)
CC--FINALLY, THE TOTAL AVERAGE ANNUAL VARIABLE COSTS, TVC, ARE
CC--COMPUTED FOR A GIVEN SET OF VALUES OF SW, QP, AND QR.
      TVC=A*APO+A2*ARI+HI*C3*EOH+LAM*EBO
      RETURN
      END
C
      SUBROUTINE TWBO(SW,QP,QR,ZB,EBO,POUT)
CC--THIS IS THE SUBROUTINE WHICH COMPUTES EBO, THE EXPECTED
CC--NUMBER OF BACKORDERS AT ANY INSTANT OT TIME, AND POUT, THE
CC--PROBABILITY OF BEING OUT OF STOCK AN ANY INSTANT OF TIME, FOR
CC--AN ITEM GIVEN SW AND QP AND QR.  NOTE THAT ARGUMENTS FOR
CC--ALPHA, GAMMA, AND DELTA ARE ONE UNIT MORE THAN THE FORMULAS IN
CC--REFERENCE [6] BECAUSE AN ARGUMENT OF "0" CAN'T BE HANDLED
CC--IN FORTRAN.
      INTEGER SW,QP,QR,X1,X2,XMAX,X
      REAL ALPHA(1000),GAMMA(1000),DELTA(1000),ZB,POUT,B1,EBO
      REAL P0,P1,P2,P3
      X1=MIN(QP,QR)-1
      XMAX=QP+QR-2
      X2=XMAX-X1
      MM=1
      X=SW+1
    2 IF(ZB.GT.50.)CALL CDFN(X,ZB,P0,P1,P2,P3)
      IF(ZB.GT.50.)GO TO 3
      CALL CDFP(X,ZB,P0,P1,P2,P3)
    3 ALPHA(X+1)=ZB*P1 -REAL(X-1)*P0
      IF(ALPHA(X+1).LT.0.0)ALPHA(X+1)=0.0
      GAMMA(X+1)=(ZB**2)*P2/2.0 + ZB*P1 -REAL(X*(X-1))*P0/2.
      IF(GAMMA(X+1).LT.0.0)GAMMA(X+1)=0.0
      DELTA(X+1)=(-REAL(X**3)/3. +REAL(X**2)/2. -REAL(X)/6.)*P0
     * +ZB*P1 +(1.5*ZB**2)*P2 +(ZB**3)*P3/3.
      IF(DELTA(X+1).LT.0.0)DELTA(X+1)=0.0
      IF(X.EQ.1)GO TO 5
      IF(X.EQ.0)GO TO 6
      MM=MM+1
      IF(MM.EQ.2)THEN
       X=SW
       IF(X.LE.0)GO TO 4
       GO TO 2
      ELSE IF(MM.EQ.3)THEN
       X=SW-X1
       IF(X.LE.0)GO TO 4
       GO TO 2
      ELSE IF(MM.EQ.4)THEN
       X=SW-X1-1
       IF(X.LE.0)GO TO 4
       GO TO 2
      ELSE IF(MM.EQ.5)THEN
       X=SW-X2
       IF(X.LE.0)GO TO 4
       GO TO 2
      ELSE IF(MM.EQ.6)THEN
```

```
      X=SW-X2-1
      IF(X.LE.0)GO TO 4
      GO TO 2
     ELSE IF(MM.EQ.7)THEN
      X=SW-XMAX
      IF(X.LE.0)GO TO 4
      GO TO 2
     ELSE IF(MM.EQ.8)THEN
      X=SW-XMAX-1
      IF(X.LE.0)GO TO 4
      GO TO 2
     ELSE IF(MM.EQ.9)THEN
      X=2
      GO TO 2
     ENDIF
   4 X=1
     GO TO 2
   5 X=0
     GO TO 2
   6 DNOM=REAL(QP*QR)
CC--PARTJ STANDS FOR PIECES OF B1 AND POUTJ STANDS FOR PIECES OF
CC--POUT.  PART1 AND POUT1 ARE COMMON FOR ALL VALUES OF SW.
     PART1=-DELTA(SW+2)+GAMMA(SW+2)+ZB*GAMMA(SW+1)
     POUT1=-GAMMA(SW+2)+ZB*ALPHA(SW+1)
     IF(SW.GT.X1)GO TO 20

CC--THIS SECTION IS FOR SW BETWEEN ZERO AND X1.

     PART2=REAL(X1*(X1+1)*(2*X1+1))/6.-REAL(SW*(SW-1)*(2*SW-1))/6.
     PART3=-(REAL(X1)+ZB)*REAL(X1*(X1+1))/2.
    * +REAL((X1+1)*X2*(X2+1))/2.+(ZB-1.)*REAL(SW*(SW-1))/2.
     PART4=-REAL(XMAX*(XMAX+1)*(2*XMAX+1))/6.
    * +REAL(X2*(X2+1)*(2*X2+1))/6.
     PART5=(REAL(XMAX)+1.+ZB)*REAL(XMAX*(XMAX+1)-X2*(X2+1))/2.
    * +DELTA(2)+GAMMA(2)+REAL(XMAX)*ALPHA(2)-ZB*GAMMA(1)
    * -ZB*ALPHA(1)
     POUT2=REAL(X1*(X1+1)-SW*(SW-1))/2.-(ZB-1.)*REAL(X1-MAX(0,(SW-1)))
    * +REAL((X1+1)*(X2-X1))
     POUT3=REAL(X1)*(REAL(XMAX)+1.+ZB)-REAL(XMAX*(XMAX+1)
    * -X2*(X2+1))/2.+ALPHA(2)+GAMMA(2)-ZB*ALPHA(1)
     POUT=(POUT1+POUT2+POUT3)/DNOM
     IF(POUT.GT.1.0)POUT=1.0
     B1=(PART1+PART2+PART3+PART4+PART5)/DNOM
     EBO=B1-REAL(SW)*POUT
     IF(EBO.LT.0.0)EBO=0.0
     GO TO 90

CC--THIS SECTION IS FOR SW BETWEEN X1+1 AND X2.

  20 PART2=DELTA(SW-X1+1)+REAL(X1)*GAMMA(SW-X1+1)
    * -ZB*GAMMA(SW-X1)-ZB*REAL(X1+1)*ALPHA(SW-X1)
     POUT2=GAMMA(SW-X1+1)-ZB*ALPHA(SW-X1)
     IF(SW.GT.X2)GO TO 30
```

```
      PART3=REAL((X1+1)*(X2*(X2+1)-SW*(SW-1)))/2.
      PART4=-REAL(XMAX*(XMAX+1)*(2*XMAX+1))/6.
     *+REAL(X2*(X2+1)*(2*X2+1))/6.
      PART5=(REAL(XMAX)+1.+ZB)*REAL(XMAX*(XMAX+1)-X2*(X2+1))/2.
     *-REAL(X1-1)*GAMMA(2)+ZB*REAL(X1)*ALPHA(1)+REAL(XMAX)*ALPHA(2)
      POUT3=REAL(X1)*(REAL(XMAX)+1.+ZB)-REAL(XMAX*(XMAX+1)
     *-X2*(X2+1))/2.
      POUT4=REAL((X1+1)*(X2-SW+1))+ALPHA(2)
      POUT=(POUT1+POUT2+POUT3+POUT4)/DNOM
      IF(POUT.GT.1.0)POUT=1.0
      B1=(PART1+PART2+PART3+PART4+PART5)/DNOM
      EBO=B1-REAL(SW)*POUT
      IF(EBO.LT.0.0)EBO=0.0
      GO TO 90

CC--THIS SECTION IS FOR SW BETWEEN X2+1 AND XMAX.

   30 PART3=DELTA(SW-X2+1)+REAL(X2)*GAMMA(SW-X2+1)
     *-ZB*GAMMA(SW-X2)-REAL(X2+1)*ZB*ALPHA(SW-X2)
      POUT3=GAMMA(SW-X2+1)-ZB*ALPHA(SW-X2)
      IF(SW.GT.XMAX)GO TO 40
      PART4=-REAL(XMAX*(XMAX+1)*(2*XMAX+1))/6.
     *+REAL(SW*(SW-1)*(2*SW-1))/6.
      PART5=(REAL(XMAX)+1.+ZB)*REAL(XMAX*(XMAX+1)-SW*(SW-1))/2.
     *-DELTA(2)-REAL(XMAX-1)*GAMMA(2)+REAL(XMAX)*ALPHA(2)
     *+ZB*GAMMA(1)+ZB*REAL(XMAX+1)*ALPHA(1)
      POUT4=(REAL(XMAX+1)+ZB)*REAL(XMAX-SW+1)-REAL(XMAX*(XMAX+1)
     *-SW*(SW-1))/2.-GAMMA(2)+ALPHA(2)+ZB*ALPHA(1)
      POUT=(POUT1+POUT2+POUT3+POUT4)/DNOM
      B1=(PART1+PART2+PART3+PART4+PART5)/DNOM
      EBO=B1-REAL(SW)*POUT
      IF(EBO.LT.0.0)EBO=0.0
      GO TO 90

CC--THIS SECTION IS FOR SW GREATER THAN XMAX.

   40 PART4=-DELTA(SW-XMAX+1)-REAL(XMAX-1)*GAMMA(SW-XMAX+1)
     *+REAL(XMAX)*ALPHA(SW-XMAX+1)+ZB*GAMMA(SW-XMAX)
     *+ZB*REAL(XMAX+1)*ALPHA(SW-XMAX)
      POUT4=-GAMMA(SW-XMAX+1)+ALPHA(SW-XMAX+1)+ZB*ALPHA(SW-XMAX)
      POUT=(POUT1+POUT2+POUT3+POUT4)/DNOM
      IF(POUT.GE.1.0)POUT=1.0
      B1=(PART1+PART2+PART3+PART4)/DNOM
      EBO=B1-REAL(SW)*POUT
      IF(EBO.LT.0.0)EBO=0.0
   90 RETURN
      END
C
      SUBROUTINE CDFP(X,Z,P0,P1,P2,P3)
CC-THIS SUBROUTINE CALCULATES THE POISSON PROBABILITIES FOR VARIOUS
CC-X VALUES AND THE COMPLEMENTARY CUMULATIVE DISTRIBUTION FUNCTION
CC-FOR THESE X VALUES (I.E.,GREATER THAN OR EQUAL TO X).
      INTEGER X
```

```
      REAL*8 P(1000),CP(1000),ZZ
      REAL P0,P1,P2,P3,Z
      ZZ=Z
      I=1
      P(I)=DEXP(-ZZ)
      CP(I)=P(I)
      IF((X-1).LT.0)GO TO 3
      IF((X-1).EQ.0)THEN
        P0=1.0-P(1)
        GO TO 4
      ENDIF
      N=X
      DO 2 I=2,N
      P(I)=ZZ*P(I-1)/REAL(I-1)
    2 CP(I)=CP(I-1)+P(I)
      P0=1.-CP(N)
CC--P0 IS THE COMPLEMENTARY CUMULATIVE DISTRIBUTION FUNCTION FOR X.
      IF((X-2).EQ.0)THEN
        P1=1.0-P(1)
        GO TO 5
      ENDIF
      IF((X-2).GT.0)P1=1.0-CP(N-1)
CC--P1 IS THE COMPLEMENTARY CUMULATIVE DISTRIBUTION FUNCTION FOR X-1.
      IF((X-3).EQ.0)THEN
        P2=1.0-P(1)
        GO TO 6
      ENDIF
      IF((X-3).GT.0)P2=1.0-CP(N-2)
CC--P2 IS THE COMPLEMENTARY CUMULATIVE DISTRIBUTION FUNCTION FOR X-2.
      IF((X-4).EQ.0)THEN
        P3=1.0-P(1)
        GO TO 7
      ENDIF
      IF((X-4).GT.0)P3=1.0-CP(N-3)
CC--P3 IS THE COMPLEMENTARY CUMULATIVE DISTRIBUTION FUNCTION FOR X-3.
      GO TO 7
    3 P0=1.0
    4 P1=1.0
    5 P2=1.0
    6 P3=1.0
    7 RETURN
      END
C
      SUBROUTINE CDFN(X,Z,P0,P1,P2,P3)
CC--THIS SUBROUTINE CALCULATES THE NORMAL COMPLEMENTARY CUMULATIVE
CC--DISTRIBUTION FUNCTION FOR VARIOUS X VALUES (I.E.,GREATER
CC--THAN OR EQUAL TO X).
      INTEGER X,NOUT
      REAL P0,P1,P2,P3,Z,ANORDF,Y0,Y1,Y2,Y3
      EXTERNAL ANORDF,UMACH
      CALL UMACH(2,NOUT)
      Y0=(REAL(X)-1.-Z+0.5)/SQRT(Z)
      P0=1.0-ANORDF(Y0)
```

```
CC--P0 IS THE COMPLEMENTARY CUMULATIVE DISTRIBUTION FUNCTION FOR X.
      Y1=(REAL(X)-Z-1.5)/SQRT(Z)
      P1=1.-ANORDF(Y1)
CC--P1 IS THE COMPLEMENTARY CUMULATIVE DISTRIBUTION FUNCTION FOR X-1.
      Y2=(REAL(X)-Z-2.5)/SQRT(Z)
      P2=1.-ANORDF(Y2)
CC--P2 IS THE COMPLEMENTARY CUMULATIVE DISTRIBUTION FUNCTION FOR X-2.
      Y3=(REAL(X)-Z-3.5)/SQRT(Z)
      P3=1.-ANORDF(Y3)
CC--P3 IS THE COMPLEMENTARY CUMULATIVE DISTRIBUTION FUNCTION FOR X-3.
      IF((X-1).LT.0)GO TO 2
      IF((X-1).EQ.0)GO TO 3
      IF((X-2).EQ.0)GO TO 4
      IF((X-3).EQ.0)GO TO 5
      GO TO 6
    2 P0=1.0
    3 P1=1.0
    4 P2=1.0
    5 P3=1.0
    6 RETURN
      END
```

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